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MODERN MATHEMATICS

MODERN MATHEMATICS

AN ELEMENTARY COURSE

BY
R. SCHORLING
ASSOCIATE PROFESSOR OF EDUCATION
UNIVERSITY OF MICHIGAN
JOHN P. CLARK
TEACHERS' COLLEGE COLUMBIA UNIVERSITY
AND
H. W. CARTER M.A. (OXON.)
HEADMASTER OF MARLING SCHOOL STROUD
AUTHOR OF "MENSURATION FOR MIDDLE FORMS"

GEORGE G. HARRAP & CO. LTD.
LONDON BOMBAY SYDNEY

First published June 1926
by George G. Harrap & Co. Ltd.
182 *High Holborn, London, W.C.*1
Reprinted June 1935

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Made in Great Britain. Printed by R. & R. Clark, Limited
Edinburgh

PREFACE

In the teaching of mathematics in the past there was a tendency to stress unduly mechanical processes which called for little real thought. Modern mathematical teaching aims at making the pupil think for himself: it seeks to arouse in the pupil a desire to find out a truth, and leads him to discover how to use that truth in the affairs of everyday life. Mechanical processes cannot, of course, be dispensed with, but they can be given their right use as a means of securing accuracy and as a means of reaching a definite and useful goal. It is particularly important that the beginner should not get the idea that mathematics is bounded by the four walls of the classroom. It is hoped that pupils who use this book will be brought to realize that mathematics directly or indirectly enters into all human activities.

The authors trust that the book will be found useful both in elementary schools and in the lower forms of secondary schools.

H. W. C.

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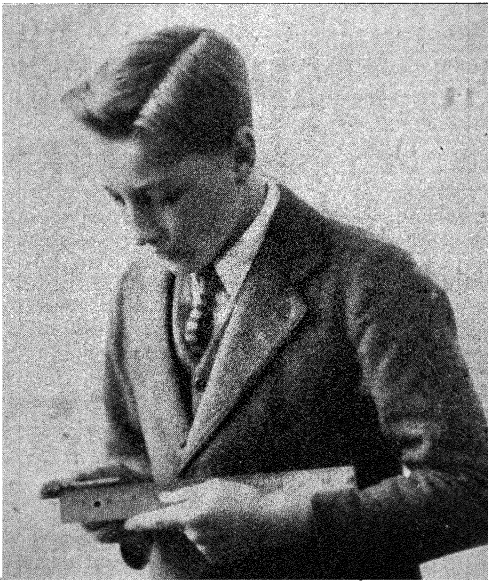
MODERN MATHEMATICS

CHAPTER I

MAKING ACCURATE MEASUREMENTS

1. Making Careful Measurements with the Ruler.— Jack Brown has to order a new bolt for his father's motor-car. The catalogue tells him that the bolts are made in three lengths, $1\frac{1}{2}$ in., 2 in., and $2\frac{1}{2}$ in.



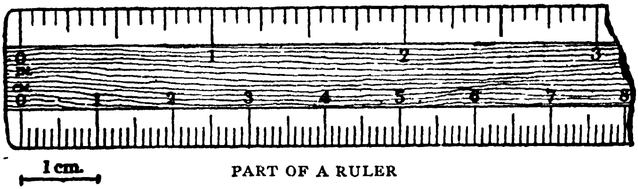
JACK BROWN MEASURING A BOLT

The picture shows how Jack measures the length of an old bolt so that he can send for a new one like it.

EXERCISES IA

1. Can you read the marks on the ruler shown in the picture on p. 9? What do those marks show?
2. How long a bolt should Jack order?

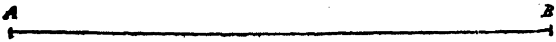
2. The Most Common Instrument for Measuring—the Ruler.—As you do the following exercises study the divisions on the upper edge of the ruler in the illustration below. It will be even more helpful if you have before you a ruler like the one shown here.



PART OF A RULER

EXERCISES IB

1. What is the length of one of the principal divisions on the upper edge of the ruler? What is the length of one of the smallest divisions? Where is the zero on this ruler?
2. If the ruler is placed along AB, with the zero at A, at what point do you find the reading for the length of AB?



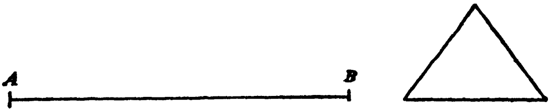
3. Find the length of the line AB, measuring it as accurately as you can.

4. Study your ruler, and complete each of the following statements :

(*a*) One of the largest units equals —— in.
(*b*) One of the smallest units equals —— in.
(*c*) Six of the smallest units equal —— in.
(*d*) Twelve of the smallest units equal —— in.
(*e*) Three of the largest units and eight of the smallest units equal —— in.

5. In a certain class the following results of measuring the length of a line were obtained : $2\frac{7}{8}$ in., $2\frac{14}{16}$ in., $2\frac{15}{16}$ in., $2\frac{13}{16}$ in., $2\frac{31}{32}$ in., $2\frac{3}{4}$ in., and $2\frac{30}{32}$ in. Arrange these results in order, putting the largest first, then the next largest, and so on.

6. What is the length of this page ? Compare the result with the results obtained by others in your class.



7. In this drawing is the line marked AB long enough to go round the triangle ?

8. Measure the width of your desk.

9. What causes errors in your measurements ? How could you reduce the amount of your errors ?

3. Importance of Measurement in Daily Life—Measurement replaces Guesswork.—The ability to make accurate measurements is all - important. Without this ability a man could not build a house, or lay a railway track, or construct an aeroplane, or make a watch. The "pinch of salt," "speck of pepper," "handful of rice," "sweeten to taste" methods of indicating amounts are being replaced by the use of precise, definite measures.

EXERCISES IC

1. Mrs A. telephoned to the greengrocer and asked him to send her a basket of strawberries. Mrs B. ordered a shillingsworth of strawberries. Mrs C. asked for a pound of strawberries. Which woman was probably the most careful buyer? Why?

2. Some people think that eggs should be sold by the pound rather than by the dozen. What arguments are there in favour of such a plan?

3. Why is it advisable for every family to have a set of weights and measures by which purchases may be checked?

4. Say which of the following measuring instruments you would select for home use: weighing scales; liquid measures, such as the half-pint, pint, and quart; dry measures, such as the quart, gallon, peck, and bushel; or linear measures, such as the foot rule, two-foot rule, yard measure, and metre stick.

5. Are bananas sold by the dozen or by the pound? Which is the fairer way?

6. Name some instruments used in measuring length?

7. Assume that your class needs to lay out a football field or a tennis court. Would there be any objection to using the foot of some member of your class as a standard measuring unit?

8. It is common for primitive peoples to use the length of the foot or the arm of some member of the tribe as a unit for measuring. Would a long foot do as well as a short one?

9. The yard is supposed to have been the length of the arm of King Henry I. Why did the people choose the King's arm as a measure of length ?

10. What two things would need to be true about a unit to make it a *standard* unit ?

11. A unit is standardized when it is *defined* and *accepted*. Explain the meaning of this statement.

12. Name some standard units of length in common use.

NOTE.—Most of the units with which you are familiar have had a long history. Many of the uncivilized tribes and nearly all the nations of the world have used a linear unit based upon the length of the human foot. They gave it a name which was derived from their word for 'foot.' In Palestine and Egypt, however, the linear unit was the length of a man's arm from elbow to finger-tip It was called a *cubit*.

4. Where Accurate Measurements are Necessary.—

Among those who measure with a high degree of accuracy are :

(*a*) *Engineers*.—In making ball bearings, such as you may have seen in motor-car wheels, the motor engineer must take great care to see that the balls have a certain size. If he makes an error of more than *three* parts in *one thousand* in making the balls they cannot be used.

(*b*) *Surveyors*.—Surveyors frequently have to survey long lines, sometimes several hundred miles long. They are not satisfied with their work if the error in the measurement is more than *one* part in *one million*.

(*c*) *Chemists*.—People who weigh very small amounts are not content with the result if the error is more than *one* part in *fifty thousand*. Their instruments are so sensitive that even a small particle of dust would change the balance.

EXERCISES ID

1. Tom Long made an error of 1 in. in measuring a line 4 ft. 2 in. long. Was the amount of the error as much as 1 part in 100 ?

2. Two pupils measured the length of a pole. Their result was 8 ft. 1 in., but the true height was 8 ft. 4 in. Their error was —— in. This was —— parts in 100. The amount of the error was what decimal part of the true length ?

3. On rather inaccurate scales a baby boy weighed $9\frac{1}{2}$ lb. On very accurate scales he weighed 9 lb. 6 oz. The error on the first scales amounted to —— oz. This amount was what part of his true weight ? How serious was this error ?

4. What errors may be involved when you measure the width of your desk ?

5. A boy measured the length of a blackboard with a two-foot rule. The other boys in the class observed him and made the following record :

(*a*) He did not always keep the edge of the rule on the edge of the blackboard.

(*b*) He let the rule slip a little, so that the second position did not begin at the exact place where the first ended.

(*c*) The rule was not straight.

(*d*) One end of the rule was worn.

(*e*) He did not measure in a straight line.

Read the preceding record until you can state the five errors that were involved in the boy's work.

6. If you were measuring with a two-foot rule which was originally correct in length, but which is now not straight, would the result be too large or too small ?

7. Yard tapes are sometimes used for measuring lengths. What error is probable when the tape gets wet ?

8. How is steel tape affected by heat? By cold?

9. Is a measurement ever likely to be absolutely correct, or is it always an approximation?

5. The Relation of Measurement to the Origin of Mathematics.—No wise man invented mathematics; it came naturally even to the earliest peoples, who found that they had often to answer these questions: (*a*) How many? (*b*) How much? (*c*) Where?

Let us first consider our own everyday life. When there is a coal strike people ask *how many* tons of coal are in hand, *how much* coal they can have, and *how many* days the strike will last. These questions can be answered only by counting. *Counting leads to arithmetic.*

Then, in building a house you want to know how long it is to be, how wide, and how high. The only way in which you can answer these questions is by measuring. *Measuring leads to geometry.*

Arithmetic and geometry are the foundation of mathematics.

In everyday life you constantly need mathematics. When you get up you reckon how long it will take you to get to school, or how long it takes you to get dressed. On your way to school you have to buy something, and you have to know how much it costs. You have to pay for it and see that you always get the correct change. You are often asked how far you live from school, and you can answer only by measuring the distance, or by letting some one else measure it.

Now, it is easy to see that even primitive people had needs relating to food, clothing, and shelter. These needs involved counting and measuring. Therefore mathematics had a very natural origin.

EXERCISES IE

1. What does a farmer do when he wants to find out how many sheep he has? When he wants to find out how much land he has? When he wants to find out where some place is situated?

2. Two Boy Scouts wanted to measure the distance between two places. Their measuring stick was not long enough to reach from one place to the other. Could they measure the distance? How?

3. Mark two points on the floor. Have the distance between them greater than the length of your ruler. Then measure with your ruler the distance between the two points.

4. What help would a piece of string have been in measuring the distance between the two points in the previous question?

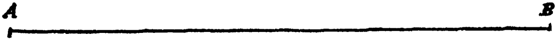
5. The preceding questions show that to find the distance between two points you need to measure along a —— line.

6. How many straight lines can you draw through two points? How many straight lines can you draw through one point?

7. A straight line is the shortest —— between two points.

6. A New Way of talking about Lines.—Measure the line shown in the figure on p. 17, AB. Where did you begin measuring? Where does the line end?

Any line that we measure must have a definite beginning point and a definite ending point. How are these points shown in this figure?



A FINITE STRAIGHT LINE

A particular length of line is called a finite straight line. We usually think of a line as indefinite, because it may be extended in either direction. But in measuring we deal with the part of a line between two 'end' points.

EXERCISES IF

1. On a line of indefinite length mark off a part AB, a part BD, and a part AC. See how many parts you can now find.

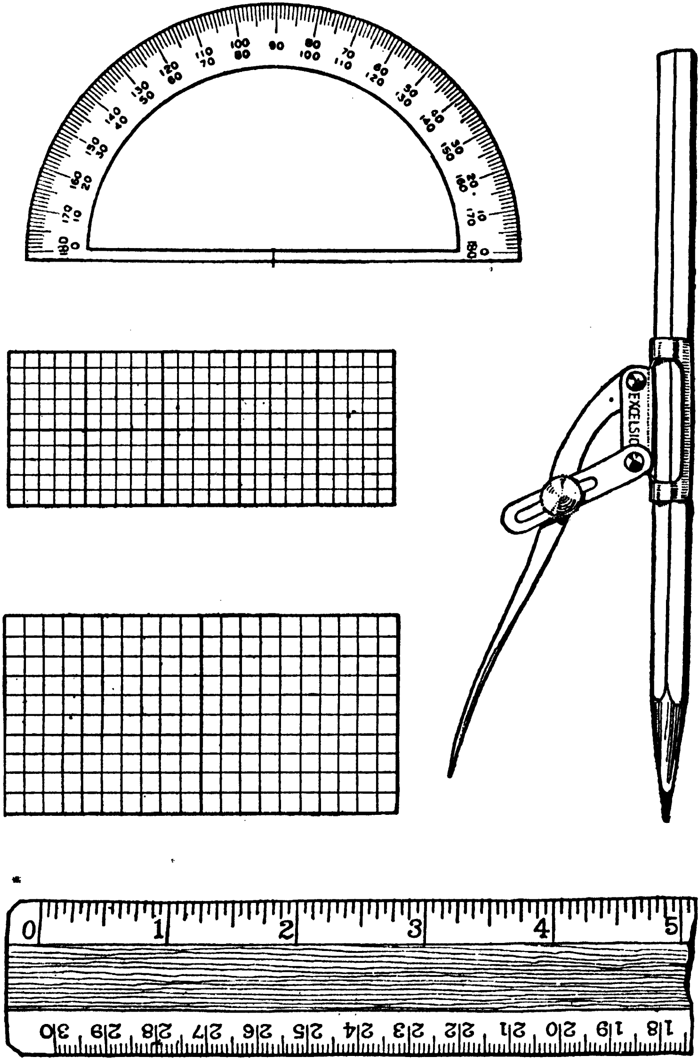
2. Lay down a line AB 4½ in. long. Mark off a part AC 2½ in., a part AD 3 in., a part AE 4 in. See how many parts you can now find, and give the length of each.

3. Which is the more convenient for measuring other lines, a line an inch long or a line a yard long?

4. Measure the line AB in the figure above. Compare your result with the results obtained by others in your class. Do all the results agree? Are all correct?

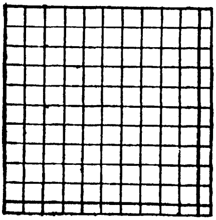
5. Lay down a line AB 5¼ in. long. Mark off a part AC 1¼ in., a part AD 2 in., a part AE 3 in., a part BF 1¾ in., a part BG ½ in. See how many parts you can now find, and give the length of each.

7. How Squared Paper is used in Measuring.— On the following page you see a number of instruments and materials that are commonly used



MEASURING INSTRUMENTS

in making accurate measurements. Two of these are samples of squared paper. It can be bought in various sizes and ruled in different ways.

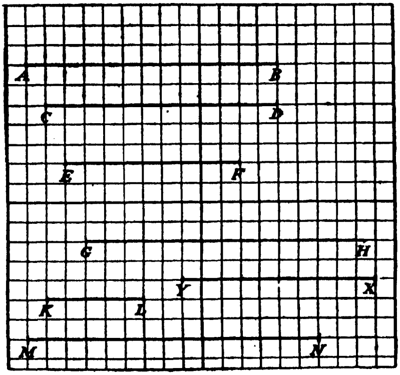


EXERCISES IG

1. What is the length of the smallest division in this illustration? Which lines are heavy? Why?

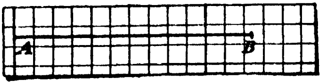
2. The next figure shows seven lines drawn on squared paper. Without using a ruler find the length of each line. Record your results like this :

AB = —— in.
CD = —— in.
EF = —— in.
GH = —— in.
YX = —— in.
KL = —— in.
MN = —— in.



3. On squared paper of the kind shown here draw lines of the following lengths : 1·2 in., 1·8 in., 1·65 in., 0·8 in., and 1·40 in. Did you need to use your ruler in drawing these lines? How does the paper serve as a ruler?

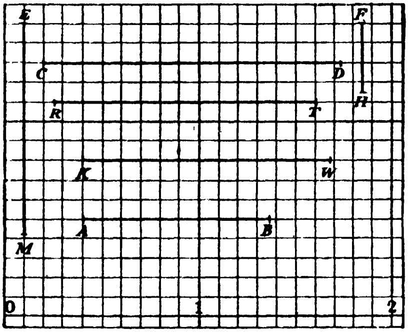
4. How long is the line AB in this figure?



We can see that the length is greater than 1·2 in., but less than 1·3 in. Our problem, then, is to estimate, as nearly as we can, the length of the small part beyond 1·2 in. It is a little less than one-half a small space. If we imagined the small space divided into 10 parts we should probably call it 4 tenths, or 0·4, of a small unit. Now, a small unit equals one-tenth of an inch. Hence, we must add 0·4 of 0·1 of an inch, which is 0·04 of an inch. This makes AB equal to 1·2 in. + 0·04 in., or 1·24 in.

Note that we have expressed the length of AB to the hundredth part of an inch.

5. Write down the length of each of the lines in this figure. Try to express the result to the hundredth part of an inch. Record your results as follows :



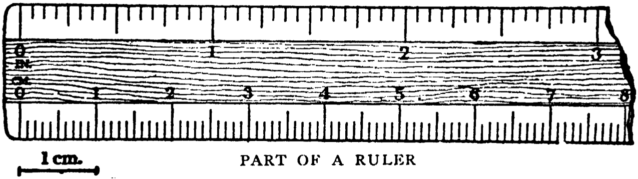
CD = —— in.
RT = —— in.
KW = —— in.
AB = —— in.
EM = —— in.
FH = —— in.

6. On squared paper draw a line AB = 1·85 in., CD = 0·58 in., XY = 1·05 in., RS = 1·30 in., MN = 2·22 in.

8. Another Measuring System—the Metric System.—The history of mathematics tells us that there were many different kinds of measuring units in use in Europe before the nineteenth century. To measure length, for example, one nation developed one kind of unit, a different nation another kind of unit. The people of France saw the need of getting a system so simple that all nations might be tempted to use it. At the time of the

French Revolution they tried to solve this problem by establishing by law the present metric system. The scientists who thought out this system agreed to use one ten-millionth part of the distance from the Equator to the North Pole as the *standard unit of length* (the *metre*). They marked this off on a metal bar which is now kept at Sèvres (near Paris).

The metric system is generally used in the scientific work of all countries. Many industries and commercial firms find it necessary to use this system in their foreign trade. Hence many people argue that Great Britain should by law adopt the metric system.



PART OF A RULER

Look at the lower edge of the ruler shown here. One of the large divisions is called a *centimetre*. (The abbreviation for centimetre is *cm.*) The smallest division on the ruler is called a *millimetre*. (The abbreviation for millimetre is *mm.*) 10 centimetres make a *decimetre* (*dm.*) 10 decimetres make a *metre* (*m.*) A metre is 39·37 in. long, or a little longer than a yard.

It is very important to know readily the relation between these units. Memorize the following:

10 mm. equals 1 cm. (nearly $\frac{2}{5}$ in.).
10 cm. equals 1 dm. (nearly 4 in.).
10 dm. equals 1 m. (39·37 in., or nearly $3\frac{1}{3}$ ft.).

Exercises Ih

1. With the metric scale on your ruler measure the length of this page. Express your result in centimetres and a decimal part of a centimetre.

2. Turn the ruler over so that you cannot see the metric scale, and draw by estimate a line 9 cm. long. Measure the line, and compare your error with the errors of others in your class. If your error is less than one centimetre you may consider the result satisfactory.

3. Draw on the blackboard a line which you think is 20 cm. long. Before measuring the line find out how many of the others in your class think you are correct within 2 cm. Then measure the line.

4. Draw a line exactly 10 inches long. Now measure the line in centimetres. Complete : 10 inches equal —— centimetres, or 1 inch equals —— centimetres.

5. Complete the following statements about the length of lines, keeping in mind the fact that 1 inch equals 2·54 centimetres :

(*a*) 25·4 cm. = —— in.
(*b*) 12 in. = —— cm.
(*c*) 76·2 cm. = —— in.
(*d*) 6·35 cm. = —— in.
(*e*) 100 cm. = —— in.
(*f*) 50 cm. = —— ft.

6. Complete these tables of *linear* measure in the metric system. (Read the first part of (*a*), then the first part of (*b*), etc.)

(*a*) 10 mm. = —— cm.
10 cm. = —— dm.
10 dm. = —— m.

(*b*) 1 mm. = —— cm.
1 cm. = —— dm.
1 dm. = —— m.

7. Complete these tables of linear measure in our system :

(*a*) 1 ft. = —— in.
1 yd. = —— ft.
1 yd. = —— in.
1 rod = —— ft.

(*b*) 1 in. = —— ft.
1 ft. = —— yd.
1 in. = —— yd.
1 ft. = —— rod.

1 rod = —— yd.
1 mile = —— ft.
1 mile = —— rods.
1 yd. = —— rod.
1 ft. = —— mile.
1 rod = —— mile.

8. Look back at the two previous questions. Then think which tables of measure, the metric or the English, would be the more difficult for a pupil who had never learned either.

9. Can you tell why the metric scale is called a *decimal* scale ?

10. Find the number of metres in 1 mile.

11. Complete the following : 1 ft. = —— cm. ; 1 cm. = —— in. ; 1 yd. = —— cm

9. How to use the Compasses in making Measurements.—A pair of compasses is a very helpful instrument in measuring straight lines.

The following directions tell you how to measure a line, such as the line LM shown here, with the compasses. Read these directions carefully.

(*a*) Place the points of the compasses on points L and M.

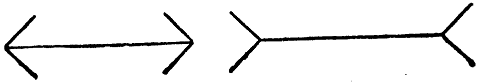
(*b*) Turn the screw which clamps the legs of the compasses. (Some compasses fasten without the screw.)

(*c*) Take up the compasses and place the points on the division marks of the ruler.

(*d*) Read the number of units (inches or centimetres) between the points.

EXERCISES II

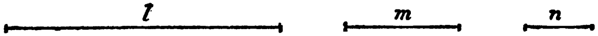
1. Look at the two straight lines shown here :



Which of them appears to be the longer? Use the compasses to find out which is the longer. Find out which is the longer by using the ruler.

2. What difficulty do you meet when you measure with your compasses a line that is longer than the 'span' of the compasses?

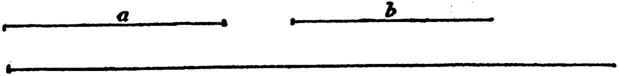
3. By the easiest method you can think of draw lines on squared paper of exactly the same length as the lines l, m, and n shown here:



4. Show that a pair of compasses is a better instrument than a ruler for transferring straight lines of given length to squared paper.

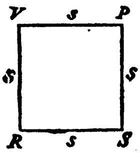
5. How could you draw a line that would be as long as the sum of the three lines in question 3?

6. On a working line, like the one shown below, construct a line that will be $a + b$ units long.



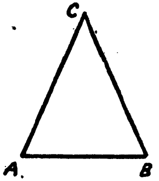
7. With your ruler measure the line obtained in the previous question. Express the result in inches. Compare it with the results obtained by others in your class.

8. The figure below is a square. On a working line draw a segment equal to the perimeter (distance round) of the square. Use your ruler to read the length of the line that you construct.



A SQUARE

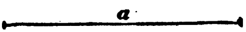
9. This figure shows an *isosceles* triangle: a triangle with two equal sides. On a working line draw a segment equal to the perimeter of the triangle.

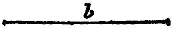


ISOSCELES TRIANGLE

Measure the result in inches, and compare it with the results obtained by others in your class.

10. Note the two lines a and b:





On a working line (or on squared paper) draw a line whose length is $2a+b$ units. Find the length of the line that you draw in centimetres.

11. Using the lines a and b shown above, draw lines whose lengths are:

(*a*) $a+2b$ units.
(*b*) $a+3b$ units.
(*c*) $2a+3b$ units.
(*d*) $3a+2b$ units.

12. Measure the line that you drew for each part of question 11. Use the metric scale. Record your results in a table like the following, after " Measured results ":

Length of line . .	$a+2b$	$a+3b$	$2a+3b$	$3a+2b$
Measured results .				
Calculated results .				
Differences . .				

13. The lines that we have been using are constructed so that a is 3·2 cm. and b is 2·3 cm. Using these numbers, calculate the result for each of the lengths in question 11. Record your results in the proper place in the table, after " Calculated results."

14. (*a*) Find the difference in each case between the measured and the calculated result. Enter the differences in your table.

(*b*) How many of your constructions are satisfactory? Call a difference of 3 millimetres or less a satisfactory result.

(*c*) Let us now say that you are 25 per cent. efficient if you had one out of four satisfactory results, 50 per cent. efficient if you had two out of four, and so on. How many in your class are 100 per cent. efficient? 75 per cent.? 50 per cent.? 25 per cent.? 0 per cent.?

15. Draw any two lines a and b. Then draw a line $a+2b$ units long. Measure, in centimetres, the line you obtain. Compare your result with the results obtained by others in the class. Complete: If some one should ask me what is the value of $a+2b$, I should say that it —— upon the —— of a and the —— of b.

16. A certain line is $2a+3b$ units long. Make a drawing to represent it. How many inches long is the line if a is $\frac{3}{4}$ in. and b is $\frac{3}{8}$ in.? What would be the length of the line $2a+3b$ if a is 2·4 in. and b is 3·5 in.? Complete: The value of $2a+3b$ depends upon the value of —— and the value of ——.

17. Which is longer: a line $2a+b$ units long or a line $b+2a$ units long? Illustrate your answer with a sketch.

10. Practical Problems involving Percentages.—Ted Baxter was reading the advertisements in the newspaper to help his father to find a good bargain in a school suit. Before he reached the advertisements for boys' suits his eye ran over advertisements for furniture, motors, jewellery, lace, suit-cases, travelling bags, Government stock, and railway stock. He was surprised to find that most of these advertisements contained phrases like the following: "marked down 5 per cent.," "discount 10 per cent. for cash," "will pay $4\frac{1}{2}$ per cent. on the investment," "will be increased 8 per cent. after January 1 next," and the like. Ted did not understand these things. His father said: "You need to learn about percentages ; everybody has to know that." Could you have helped Ted ? What is the meaning of 5 per cent. ? of 8 per cent. ?

11. The Meaning of Per Cent.—In your previous work you will have learned that 6 per cent. means 6 out of 100, or $\frac{6}{100}$; 50 per cent. means 50 out of 100, or $\frac{50}{100}$; 65 per cent. means 65 in every 100, or ·65. In answering the following questions you will learn more about the use of percentages in everyday problems.

EXERCISES IJ

1. A boy earned 8*d.* one afternoon, and spent 6*d.*, or —— per cent. of it, on sweets.

2. Smith solved 8 out of 12 problems in a test. His mark on the test was —— per cent.

3. Complete the following statements about the work of certain pupils :

(*a*) John spelled 18 out of 25 words, which is equivalent to —— out of 100, or —— per cent.

(*b*) Mary answered correctly 7 history questions out of 10, or —— out of 100, or —— per cent.

(*c*) Kathleen obtained 80 per cent. in her science test, answering —— out of 10 questions.

(*d*) William did correctly 12 out of 16 mathematics problems, or —— out of 100, or —— per cent.

(*e*) Peter spelled 9 out of 15 words, or —— out of 100, or —— per cent.

4. A pint, which is 0·5 qt., is what per cent. of a quart ?

5. A peck, which is 0·25 bu., is what per cent. of a bushel ?

6. A pint is what fraction of a gallon ? A pint is how

many hundredths of a gallon? A pint is what per cent of a gallon?

7. A sixpence is what fraction of 5*s*.? What per cent. of 5*s*. is sixpence?

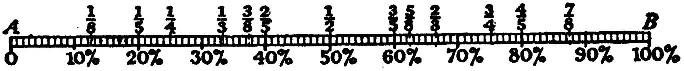
8. What is $\frac{1}{4}$ of £3? What is 0·25 of £3? What is 25 per cent. of £3?

9. On the average about 2 per cent. of the pupils in a certain school are late on Monday. Out of 1000 children in the school —— are likely to be late.

10. Out of 800 pupils 3 per cent. have been late or absent. How many pupils have not been late or absent?

12. Common Fractions, Decimals, and Percentages may all mean the Same Thing.—Every day on his way home from school Jack Dixon stopped to look at Barker's show window. He wanted a scooter for Christmas, and Barker's had some fine ones. One evening Jack said: "I see that Barker's scooters are getting cheaper every day. Yesterday he had $\frac{1}{6}$ off, and to-day it is $16\frac{2}{3}$ per cent. off." Mr Dixon said: "Jack, you are mistaken. $\frac{1}{6}$ of an amount and $16\frac{2}{3}$ per cent. of it mean the same thing. Those scooters have not been reduced since yesterday." Was he correct?

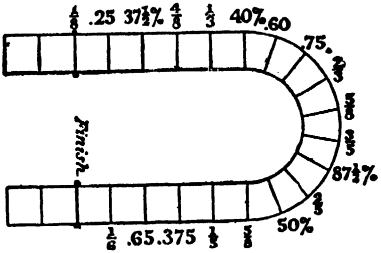
After supper Mr Dixon said he would show Jack an easy way of learning how to express fractions as percentages. He drew a line AB, and marked it off into hundredths.



By the aid of this diagram it was easy for Jack to see that $\frac{1}{2} = 50$ per cent., that $\frac{1}{4} = 25$ per cent., that $\frac{3}{4} = 75$ per cent., and that $\frac{1}{8} = 12\frac{1}{2}$ per cent.

Exercises Ik

1. See how many minutes it takes you to run round this 'percentage track.' If necessary refer to Mr Dixon's scale. If the number is given as a common fraction or as a decimal fraction express it as a percentage. If given as a percentage express it as a common fraction.



2. After practising for five evenings Jack was able to go round the track in 2 minutes. Keep *your* score.

3. Express these common fractions as percentages: $\frac{2}{3}$, $\frac{4}{5}$, $\frac{3}{8}$, $\frac{5}{12}$.

4. Express these decimals as percentages: 0·37, 0·2, 0·055.

13. Facts Every One should know.—Make a copy of the table on the next page, and see how long it takes you to fill in the missing numbers.

Exercises Il

1. Mary Brown found a purse which contained 20*s*. She returned it to the owner, who gave her 15 per cent. of the money for a reward. How much did she receive? Remember that 15 per cent. of 20*s*. means $0{\cdot}15 \times 20s$.

2. Mr Barrett sold a plot of land for a friend for £650. His friend gave him 4 per cent. of the money for making the sale. How much did Mr Barrett receive?

Common Fractions in Lowest Terms	Fraction with 100 as a Denominator	Decimal Fractions	Percentages
$\frac{1}{2}$	$\frac{50}{100}$	·50	50 %
$\frac{1}{3}$			
	$\frac{66\frac{2}{3}}{100}$		
		·25	
			75 %
		1·25	
	$\frac{37\frac{1}{2}}{100}$		
$\frac{5}{8}$			
			$87\frac{1}{2}$ %
$\frac{1}{5}$			
			40 %
		·60	
	$\frac{80}{100}$		
$\frac{1}{6}$			

3. Grandfather Jones gave £52 to his three grandchildren: 40 per cent. to Joan, 35 per cent. to Robert, and the remainder to Thomas. How much did each receive?

4. Jim Sanders bought a 40*s*. Boy Scout suit from his uncle, who gave him 35 per cent. off. How much did the suit cost Jim Sanders?

5. About 3·5 per cent. of the weight of a certain cow's milk is butter fat. How many pounds of butter fat are there in 46 lb. of milk?

6. Williams, the grocer, bought potatoes at 10*d*.. a stone. He sold them at a profit of 40 per cent., or at how much a stone?

7. A milk-dealer pays 6*d*. a quart for milk. He makes a profit of $33\frac{1}{3}$ per cent. on each quart, or ——*d*. He sells milk for ——*d*. a quart.

8. Peter Dunn bought 25*s*. worth of books from a bookseller, who allowed him to take off 10 per cent. if he paid the bill within 10 days. He saved —— by paying within 10 days.

9. Mr Jackson's electric light bill for January was 13*s*. 4*d*., with 5 per cent. off for cash within 10 days. How much would he save by paying within the 10 days?

10. (*a*) 20 per cent. of my potatoes were no good. Out of 25 bushels I had only —— bushels of good ones. (*b*) At 5*s*. a bushel, what did the 25 bushels cost me? What was that a bushel for the good ones?

11. Mildred spent only 64 per cent. of her allowance of 6*s*. 3*d*. a week, thereby saving —— per week.

12. Complete the following:

(*a*) 40% of 16*s*. 8*d*. = ——.
(*b*) 25% of 1 ft. = —— in.
(*c*) $33\frac{1}{3}$% of 1 mile = —— ft.
(*d*) 50% of 29·46 = ——.
(*e*) $37\frac{1}{2}$% of £8 = ——.
(*f*) $83\frac{1}{3}$% of £36 = ——

13. Muriel bought a dress that was marked £3 10*s*. at a reduction of 15 per cent. What did she pay for the dress?

14. Mr Farmer sold 475 pounds of milk in one week. The milk contained 4 per cent. butter fat, for which he received 1*s.* 1*d.* a pound. How many pounds of butter fat did he sell? What did he receive for it?

15. A headmaster reported that he had 650 pupils in his school during a certain year. He estimated that there would be an increase of about 12 per cent. in the number of pupils the next year. How many pupils did he expect to have?

14. How to find what Per Cent. One Number is of Another.—Two boys took a spelling test of 20 words. Brown spelled 15 words correctly. The master marked his paper "75 per cent." Was it marked correctly? 15 is what part of 20? 15 is what per cent. of 20? Robinson spelled 20 out of 25 words chosen from the same page. Was he a better speller than Brown?

You cannot answer this question unless you know how to find what per cent. one number is of another. Here is the solution:

Brown's score was $\frac{15}{20}$, or $\frac{75}{100}$, or 75 per cent.
Robinson's score was $\frac{20}{25}$, or $\frac{80}{100}$, or 80 per cent.

EXERCISES IM

1. $4\frac{1}{2}$ is what per cent. of 5?
2. 3 is what per cent. of 40?
3. 8 is —— per cent. of 15.
4. 6 is —— per cent. of 90.
5. 6 is —— per cent. of 10.
6. 2 is —— per cent. of 10? of 100? of 200? of 1000?
7. What per cent. of a yard is 12 in.? 18 in.? 20 in.? 24 in.? 30 in.?

8. A gardener raised 60 bushels of potatoes last year. This year he raised 90. This was an increase of —— bushels, or an increase of —— per cent. on last year's crop.

9. In a room of 40 pupils 26 are boys. What per cent. are boys?

10. A clerk's weekly salary is £3. He saves 16*s*. a week. What per cent. of his salary does he save?

11. If you have read 75 pages of a book of 300 pages you have read —— per cent. of the book.

12. My watch case is "16 carats fine." This means that it is $\frac{16}{24}$ pure gold. What per cent. of it is pure gold?

13. Pencils are 4*d*. each or 2 for 7*d*. If you buy 2 pencils you save —— on what would otherwise cost 8*d*. This is a saving of —— per cent.

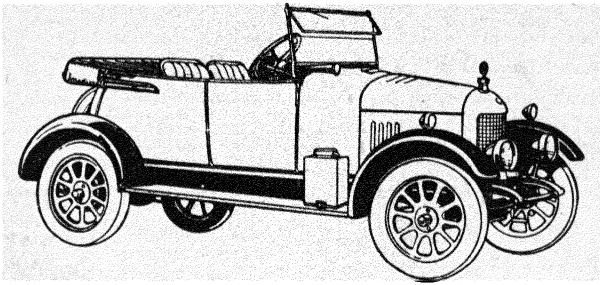
14. If 3 pupils out of 27 have been neither absent nor late for school this year, what per cent. of the pupils have a perfect attendance?

15. Mary Short received 15*s*. a week when she first began to work. At the end of the first week her weekly salary was increased to 19*s*. This was an increase of —— per cent.

16. Tom Long started to work at 15*s*. a week. He received an increase of 33⅓ per cent., or —— shillings per week increase.

17. A puzzle: Think of some number, increase it by 50 per cent. of itself, multiply the result by 2, divide the last result by 3. If you did the work correctly, you now have the number with which you started. Can you explain why this is true?

15. Cost of Running a Car.



ESTIMATE OF RUNNING COSTS FOR MORRIS-COWLEY FOUR-SEATER CAR—12,000-MILE YEAR

Actual Running Costs

Petrol: 32 miles per gallon at 1*s*. 5½*d*. a gallon.

Oil: 1 gallon for every 1000 miles at 5*s*. 6*d*. a gallon.

Set of 4 tyres and 1 tube: tyres, £3 2*s*. 3*d*. each; tube, 11*s*. 9*d*.

Standing Charges

Interest on capital (*i.e.*, cost of car, £195) at 5 per cent.

Tax: £12.

Insurance: free.

Depreciation: first half-year, 15 per cent. (cost of car, £195); second half-year, 12½ per cent. of remainder.

Repairs and overhaul after 10,000 miles: £20

EXERCISES IN

The necessary particulars for working the following questions are given above.

1. Find the cost of petrol for the year.
2. Find the cost of oil for the year.
3. Find the cost of tyres and tube for the year.
4. Find the total of the actual running costs
5. Express the above as cost per mile, giving your answer in pence correct to 3 places of decimals.

6. For working out the total standing charges, find (*a*) interest on capital, (*b*) depreciation for first half-year, (*c*) depreciation for second half-year (correct to a penny).

7. Find the total standing charges.

8. Express the above as cost per mile, giving your answer in pence correct to 3 places of decimals.

9. Find, taking into consideration the actual running costs and the standing charges, the cost per mile of running the car. Give answer in pence correct to 3 places of decimals.

REVISION EXERCISES I

1. Find some object in your classroom which you think is about 1 cm. long; another about 1 dm. long; and a third about 1 m. long.

2. Let one of the pupils in your class stand facing the class. Let each pupil estimate his height in inches. Now have the pupil's height measured and recorded on the board. If your estimate is within two inches of the measured result you have satisfactory skill in estimating.

3. Repeat the experiment in question 2, selecting a pupil of a very different height. Now estimate the pupil's height in centimetres. If your estimate is within five centimetres of the measured height it may be considered satisfactory.

4. A motor-car that cost £230 depreciated 30 per cent. of its cost price the first year, 20 per cent. of its cost price during the second year, and 10 per cent. of its cost price during the third year. What was it worth at the end of the third year?

5. A new car sold for £790; at the end of a year it sold for £720 The depreciation was —— per cent.

6. A science class had a lesson in careful measurement. Each pupil measured the length of an object. Here are the three best results:

Pupil	Result	Correct Result	Error in Centimetres
Johnson . .	$8\frac{1}{2}$ in.	22 cm.	
King . .	4 in.	11 cm.	
Roberts . .	$10\frac{1}{4}$ in.	26 cm.	

Remembering that 1 in. = 2·54 cm., find the error of each of the three pupils.

7. Look at these examples, and try to state a rule for changing common fractions to per cents. :

(*a*) $\frac{3}{8} = \cdot 375$

$= 37\frac{1}{2}$ per cent.

$$\begin{array}{r|l} 8 & 3\cdot 00 \\ \hline & \cdot 375 \end{array}$$

(*b*) $\frac{1}{16} = \cdot 0625$

$= 6\frac{1}{4}$ per cent.

$$\begin{array}{r|l} 4 & 1\cdot 00 \\ \hline 4 & \cdot 25 \\ \hline & \cdot 0625 \end{array}$$

8. Arrange these numbers in order of size, beginning with the largest : $\frac{5}{8}$, $\frac{4}{5}$, 75 per cent., 0·81, $\frac{7}{11}$, $\frac{2\frac{1}{2}}{3}$, 85 per cent. Think of each as a decimal.

9. Fred earned 55*s*. a week. He spent 60 per cent. of his salary for board and lodgings, 10 per cent. for clothes, 12 per cent. for other expenses, and saved the rest. How much did he save during a period of 15 weeks ?

10. Jane paid 30*s*. for some silk, and sold it for 42*s*. What per cent. of the cost price did she gain? What per cent. of the selling price did she gain?

11. Mr Burton bought some tools for 56*s*., less 20 per cent. discount. He sold them for 56*s*. What per cent. did he gain on the amount he paid for the tools ?

12. Dick bought a dog for £4. After spending 5*s*. for food for the dog and 10 hours' time in caring for it, he sold the dog for £6. Dick thought his time was worth 2*s*. an hour. What per cent. of the total cost did he gain or lose?

PRACTICE TESTS FOR ACCURACY AND SPEED

These practice tests are designed to help you to become more accurate and rapid in the fundamentals of mathematics. You should practise one or more of these tests daily until you can write the correct answers to each test in four minutes. Place a strip of paper below each row of examples so that it will not be necessary to copy the figures or to write the answers in the book. Your motto should be "*Accuracy* first, then speed."

You pass a test that contains 10 or more examples when you write all the answers and have not more than one error. If a test has less than 10 examples, you must do every example correctly to pass that test.

TO THE TEACHER. Careful investigation has demonstrated the value of regular *timed practice* exercises in which each pupil tries to beat his own previous achievement in his effort to attain a definite goal. Studies in the psychology of learning emphasize the great importance of having the learner become interested in his improvement. These practice exercises utilize this principle (*a*) by setting up specific goals for the pupil, (*b*) by making the pupil keep a record of his daily achievement, and (*c*) by providing each pupil with an opportunity to concentrate upon those processes which present peculiar difficulties to him. They permit pupils to progress according to their varying skills and abilities.

Teachers who have used standardized, timed practice exercises are gratified at the unusual interest which children develop.

How to give the Tests

(1) Start all pupils at the same time. Allow all the pupils exactly four minutes for each test. (2) Let pupils correct their work by referring to the answers at the back of the book. (3) Some pupils need to be taught how to keep their records. (4) The book should be inspected at intervals by the teacher to discover any pupils who may need special instruction. (5) It stimulates the class if the teacher collects each day the papers of all pupils who have succeeded in passing a test. (6) The tests

should be given at the beginning of the lesson period. (7) Frequently pupils will ask questions just before beginning a test. This is an opportune time at which to explain and give illustrations of the point in question. Similarly, pupils often want to discuss their errors after they have corrected their papers. It is profitable to discuss common errors at this time. The time given to such discussions should be brief.

Test 1. Addition and Subtraction

Add:

5	8	9	7	7	9	7	8	7	3	8	9	8	6	5	8	7
8	6	9	9	5	5	8	7	6	9	5	7	8	7	7	3	4

9	9	6	6	6	5	9	7	5	4	4	9	8	6	8	7	9
3	8	6	8	5	6	5	7	9	8	9	6	9	9	4	3	4

9	0	9	7	9	0	7	6	0	8	8	5	9	4	0	3	9
8	7	6	6	0	8	9	8	6	9	5	0	5	9	4	8	2

Subtract:

15	12	11	14	12	10	13	12	8	13	11	14	10	13	11
9	7	6	8	6	5	9	8	0	7	7	9	4	8	5

10	12	14	15	11	14	7	16	10	13	10	11	16	15	16
6	4	5	8	4	6	0	7	1	5	3	2	9	6	9

17	15	9	18	12	9	8	17	13	14	11	10	16	12	13
8	7	0	9	5	1	1	9	4	7	3	2	8	3	6

How to keep a Record of your Progress

(1) You should draw in your notebook a form for each

Record of Improvement, Test 1

Trial . .	1st	2nd	3rd	4th	5th
Number right .					

test like the one shown here. After each trial record the number of examples you did correctly.

(2) Turn to the back of this book. There you will find a sample record card. Make a copy of this record card in your notebook. When you pass a test enter the record on the record card.

Test 2. Multiplication

Multiply:

4	9	1	7	3	6	2	4	6	3	3	5	4	2	8	5	7	4	7
4	2	0	3	9	4	0	5	3	0	5	6	0	9	3	0	8	3	2

4	8	6	6	4	7	9	5	8	2	7	9	5	3	0	3	7	0	6
7	2	0	5	6	0	3	4	0	8	4	0	5	8	1	7	6	0	2

5	0	3	8	0	9	9	0	4	5	0	5	8	0	8	5	4	3	2
3	2	6	4	3	9	6	4	8	9	5	8	7	6	6	7	9	3	4

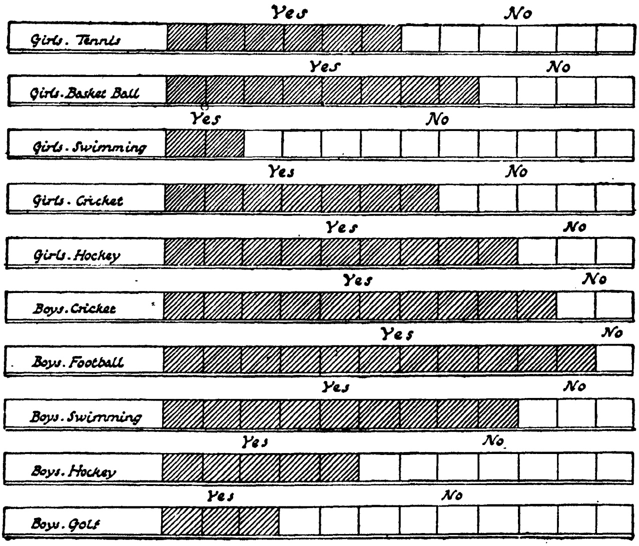
0	7	9	0	9	6	8	9	9	8	7	6	7	6	6	8	3	2	1
7	5	8	8	4	7	8	7	5	5	7	6	9	9	8	9	4	7	9

Be sure to keep your record of progress in this test on a form like the one shown for Test 1. When you pass Test 2 turn to Test 3, p. 64.

CHAPTER II

HOW GRAPHS ARE USED TO PICTURE NUMBERS

16. The headmaster of the Cromwell Secondary School for boys and girls gave a talk in the school assembly about the importance of recreation. He used a large diagram, like the one you see below, in order to show what games were played in the Cromwell School and how many boys or how many girls took part in those games :



EXERCISES IIA

1. Study the preceding diagram. What part of the girls play tennis ? What per cent. play tennis ?

2. What part of the boys play golf ? What per cent. is this ?

3. Who make the better showing in cricket ? By how much per cent. ?

4. What per cent. of the boys play football ?

5. What per cent. of the girls swim ?

6. Who make the better showing in hockey ? By how much per cent. ?

7. There are 288 girls in the Cromwell Secondary School and 264 boys. How many girls play basketball ? How many boys can swim ?

8. How many girls play tennis ? How many boys play football ?

9. Assuming that every able-bodied boy and girl should learn to swim, does your class make a better or a worse showing than the boys of the Cromwell School ?

10. Bring to class illustrations of diagrams that have been used to represent various amounts, or sizes, of things. You will find many of them in magazines and newspapers.

17. A Second Method of showing Number Relations: the Use of a Table.—In the study of a problem the most common method of stating facts is to show them in a well-arranged table.

EXERCISES IIB

1. Study the table below. It shows similar facts to those given by the diagram on p. 40. What part of the total number of girls play basket-ball ? Express this as a common fraction and as a decimal fraction (correct to two places).

TABLE OF THE GAMES PLAYED BY THE BOYS AND GIRLS IN THE SECONDARY SCHOOLS OF A CERTAIN COUNTY

——	Boys	Girls	Total
Number of pupils	1686	1773	3459
Number who play basket-ball	—	942	942
Number who play hockey	320	1123	1443
Number who play cricket	1432	621	2053
Number who play football	1570	—	1570
Number who swim	1211	492	1703

2. What percentage of the boys play football ?

3. What percentage of the girls swim ?

4. What part of the total number of girls play cricket ? Express this as a decimal fraction (correct to 2 places).

5. What part of the total number of boys play cricket ? Express this as a decimal fraction (correct to 2 places).

6. What percentage of the total number of pupils can swim ?

7. What part of the total number of pupils play cricket ? Express this as a decimal fraction (correct to 2 places).

8. What percentage of the boys do not play hockey ?

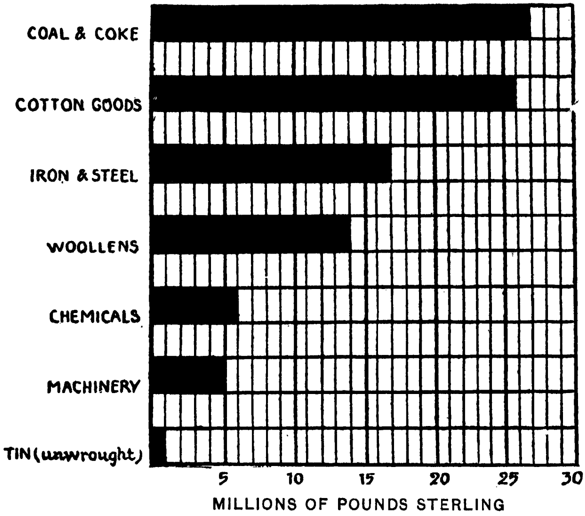
18. Bar Graphs.—A graphical representation of facts involving large or small numbers may be made by what is called a bar graph. The graph in the next exercises is a bar graph.

EXERCISES IIc

1. Study carefully the graph below, and note exactly what it illustrates.

2. Each small space along the horizontal line represents how many pounds ? Where is the scale written ?

IMPORTS OF BRITISH GOODS INTO FRANCE, 1918



3. Complete: If one bar is twice as long as another, it tells you that ——.

4. Why are the names of the imported goods printed at the left end of the bars instead of at the right?

5. Of which of the imported goods was the value the greatest? the least? Which were nearly equal in value?

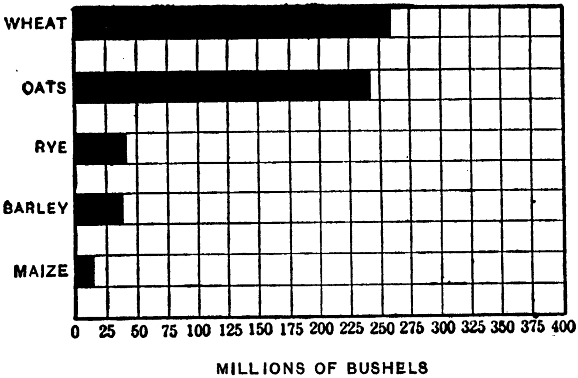
6. What was the value of each of the different classes of imported goods? Record your results in a table.

EXERCISES IID

1. What is the name of the graph on this page? Where is it written?

2. Where is the scale written? See where the scale is written in each of the other graphs appearing in this chapter.

IMPORTANT CEREALS PRODUCED IN FRANCE



3. Copy and complete : The amount of wheat produced is —— times as large as the amount of corn.

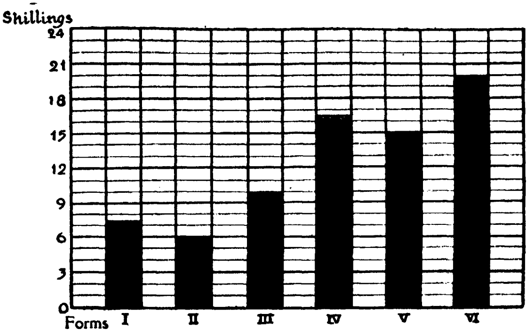
4. How long would a bar need to be to represent 120 million bushels of some product ?

5. Can you tabulate the figures (that is, arrange them in columns) on which this graph is based ? How many columns do you need ? Make the table.

6. Is the graph better than the table ? Why ?

19. The Use of Graphs in Publicity Work.—The forms in the Raleigh School decided to have a competition to see which of them could collect during the term the most money for Dr Barnardo's Homes. One of the forms made a graph each week and posted it on the school notice-board, to let the school know how much each form had got. One of these graphs is shown here.

GRAPH SHOWING MONEY RAISED (TO DATE) FOR DR BARNARDO'S HOMES



EXERCISES IIE

1. How much more had Form IV collected than Form II ?

2. How much had Form VI collected ?

3. What was the total amount collected when the above graph was posted ?

4. What per cent. of the total had Form V collected ?

5. What per cent. of the total had Form I collected ?

6. Why is it desirable to have all the bars, as here, of equal width ?

7. Why are the bars blackened ?

8. The largest amount collected was —— shillings more than the smallest amount collected.

20. Making Bar Graphs.—We have now learned how to read bar graphs. Next we shall construct some graphs. Each pupil should have graph paper ruled either in inches and tenths of an inch or in centimetres and millimetres, and should have practice with both kinds.

EXERCISES IIF

1. Six schools had an athletic contest. The list below shows the number of points scored by each school :

Raleigh	27	Cromwell	32
Tennyson	46	Salisbury	36
Wolsey	19	Milton	40

Represent these facts by a bar graph such as you would want to use if you belonged to the Tennyson School. First choose the scale. This means that you need to decide what each space shall represent in your graph. In what order will you arrange the schools ? What title will you put above the graph ?

2. The Cromwell School decided to run a cadet corps. A campaign was organized to see which of the four top forms would have the most boys in the corps. The sixth form thought that a neat-looking graph would be good publicity in the campaign. Make a graph, using their table shown here :

Form	No. of Boys in the Form	No. of Boys in the Cadet Corps
VI	18	13
V	28	17
IVA	25	11
IVB	22	7

3. Make a graph to show the increase in size in the form taking the Oxford School Certificate Examination in the Wolsey School. These are the numbers for the ten years ending with 1924 :

1915	6	1920	14
1916	6	1921	17
1917	8	1922	21
1918	9	1923	26
1919	12	1924	28

4. From a geography text-book find the five longest rivers of the world and represent the lengths of these rivers by a bar graph.

5. From a geography text-book find the five highest mountains of the world and represent the heights of these mountains by a bar graph.

6. There are 238 boys in the Hampden School. Of these 13 are under 11 years of age, 41 are between 11 and 12 years of age, 49 are between 12 and 13, 42 are between 13 and 14, 32 are between 14 and 15, 30 are between 15 and 16, 21 between 16 and 17, and 10 between 17 and 18. Represent this by a bar graph.

7. Mount Everest is 29,002 feet high. Here are some facts in connexion with the 1924 expedition to conquer the world's highest mountain: Base Camp, 16,500 ft.; Camp 5, 25,000 ft.; Camp 6, 26,700 ft.; point reached by Norton and Somervell, 28,130 ft.; point at which Mallory and Irvine were last seen, 28,230 ft. Illustrate by a bar graph.

8. Here are some figures given by the Ministry of Agriculture with reference to the crops of 1924 as compared with those of 1923:

Crop	Yield in Tons 1923	Yield in Tons 1924
Wheat . . .	1,522,000	1,373,000
Barley . . .	965,000	1,027,000
Oats . . .	1,353,000	1,514,000
Hay . . .	7,707,000	7,998,000

Illustrate by a bar graph.

21. A New Way of comparing Numbers: Ratio.—When two numbers are so related that the first is twice the second they are said to be in the ratio of 2 to 1. If the first quantity is 5 times the second they are in the ratio of 5 to 1. If the first is $1\frac{1}{4}$ times the second they are in the ratio of $1\frac{1}{4}$ to 1, or 5 to 4.

Exercises IIG

1. Study these pairs of numbers to see which pairs are in the ratio of 2 to 1, of 5 to 1, and of 5 to 4:

(*a*) 10 and 8
(*b*) 10 and 5
(*c*) 10 and 2
(*d*) 1 and $\frac{1}{2}$
(*e*) 1 and $\frac{1}{5}$
(*f*) 1 and ·8
(*g*) 12 and 6
(*h*) 15 and 3
(*i*) 15 and 12

2. Fill in the missing numbers:

(*a*) $\frac{1}{2}$ in. = —— times $\frac{1}{4}$ in.
(*b*) $\frac{3}{5}$ lb. = —— times 1·8 lb.
(*c*) 1 hr. = —— times 20 min.
(*d*) 2*s*. 6*d*. = —— times 5*d*.
(*e*) The ratio of 12 to 4 is —— to 1.
(*f*) The ratio of 24 to 4 is —— to 1.
(*g*) The ratio of 48 to 4 is —— to 1.
(*h*) The ratio of 6 to 4 is —— to 2.

3. Fred Nicholson weighs 60 pounds; his sister Dorothy weighs 48 pounds. Compare their weights.

4. Supply the missing parts in the following statements. Do this in each of three ways: (1) by using a decimal number, (2) by filling in a common fraction or a mixed number, and (3) by supplying a per cent.

(*a*) 240 ft. is —— as long as 150 ft.
(*b*) 25*s*. is —— as much as £1.
(*c*) 2*s*. 9*d*. per dozen is —— as expensive as 2*s*. per dozen for the same kind of eggs.
(*d*) 50 minutes is —— as long as 1 hr. 30 min.

You can express the relation between the quantities in each of the exercises above by using a ratio; thus:

(*e*) The ratio of 240 ft. to 150 ft. is —— to 5.
(*f*) The ratio of 25*s*. to 20*s*. is —— to 4.
(*g*) The ratio of 33*d*. to 24*d*. is 11 to ——.
(*h*) The ratio of 50 min. to 90 min. is —— to ——.

5. To find the ratio of one quantity to another express both in the *same unit* of measure before dividing. Thus:

(*a*) The ratio of 3 ft. 3 in. to 3 in. is the ratio of 39 in. to 3 in. This ratio is —— to ——.

(*b*) The ratio of 4 bu. to 4 qt. is —— to ——.

(*c*) The ratio of 1½ dozen eggs to 2 eggs is the ratio of 18 eggs to 2 eggs. This ratio is —— to ——

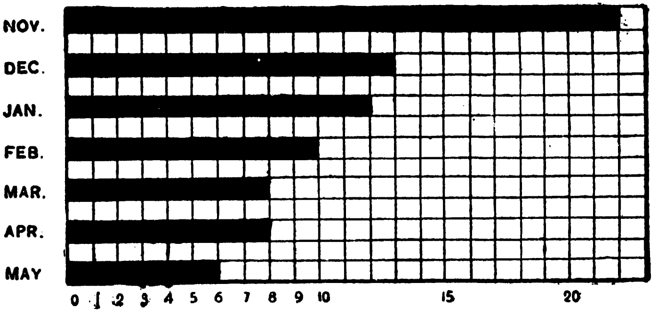
6. Supply the missing numbers in these statements:

(*a*) The ratio of 2 lb. 4 oz. to 6 oz. is —— to ——.
(*b*) The ratio of 1 ton 760 lb. to 500 lb. is —— to ——.
(*c*) £4 = —— times 16*s*.
(*d*) 3 gal. 1 pt. = —— times ¼ pint.

7. Dorothy used the following recipe for chocolate: "6 cups of sugar, 1½ cups of milk, 5 oz. chocolate." She wanted to use only 4 cups of sugar. How much of each of the other articles should she use?

8. The forms in the Drake School had a competition to decrease lateness. The graph shows the total number of 'lates' for one whole class by months.

NUMBER OF PUPILS LATE



Complete this table:

Month . .	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May
Number of 'Lates'	22						

D

Find the missing numbers :

(*a*) The ratio of the number of January 'lates' to the number of May 'lates' is —— to ——.

(*b*) There were 12 'lates' in January compared with 8 in March, or there were 3 in January to every —— in March, or the ratio of the number in January to the number in March is —— to ——.

(*c*) Find the ratio of the number of 'lates' in November to the number in each of the other months. Record your results in a table.

9. When the quantity of flour in a recipe is three times the quantity of water we say that flour and water are mixed in the ratio of 3 to 1.

(*a*) If you use 12 cups of flour you must use —— cups of water.

(*b*) If you use 2 cups of water you must use —— cups of flour.

(*c*) If you use $5\frac{1}{3}$ cups of water you must use —— of flour.

(*d*) If you use $7\frac{1}{2}$ cups of flour you must use —— cups of water.

10. The following recipes are taken from a cookery book :

Dumplings

1 pound of beef suet
$2\frac{1}{2}$ cups of baking flour
6 teaspoonfuls of baking powder
2 teaspoonfuls of salt
1 teaspoonful of pepper
2 teaspoonfuls of parsley
$\frac{1}{2}$ cup of cold water

Angel Cake

3 cups of whites of eggs
3 cups of sugar
2 teaspoonfuls of cream of tartar
2 cups of flour
$\frac{1}{2}$ teaspoonful of salt
2 teaspoonfuls of vanilla

(*a*) How much would you use of each article to make $\frac{1}{2}$ as large a quantity ?

(*b*) To make $\frac{2}{3}$ as much ?

(*c*) To make $1\frac{1}{3}$ times as much ?

11. Complete the following:

(*a*) The ratio of 2 to 5 equals the ratio of —— to 100. Thus 2 is —— per cent. of 5.

(*b*) The ratio of 4 to 5 equals the ratio of —— to 100. Thus 4 is —— per cent. of 5.

(*c*) The ratio of 5 to 4 equals the ratio of —— to 100. Thus 5 is —— per cent. of 4.

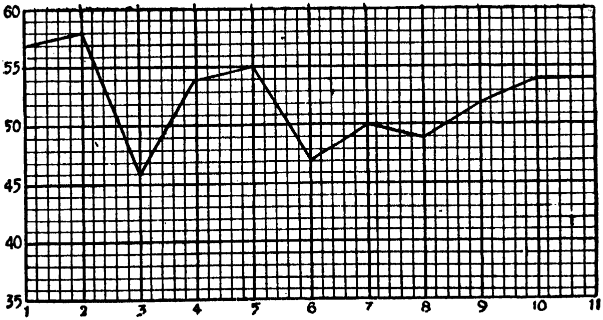
22. The Line Graph.—Form IIA in the Faraday School was studying facts about temperature. The following record was kept:

TABLE SHOWING MIDDAY TEMPERATURE

November . .	1	2	3	4	5	6	7	8	9	10	11
Temperature (F.) .	57°	58°	46°	54°	55°	47°	50°	49°	52°	54°	54°

At the end of the 11-day period the facts contained in the table were represented by a graph, as shown below:

A TEMPERATURE GRAPH FOR THE FIRST ELEVEN DAYS IN NOVEMBER



EXERCISES IIH

1. What does the table tell? What does the graph show?

2. Which method of showing the changes in temperature in November do you prefer?

3. What two scales are used in the graph? Between which dates did the temperature increase? How are decreases shown on the graph? What was the temperature on the second day?

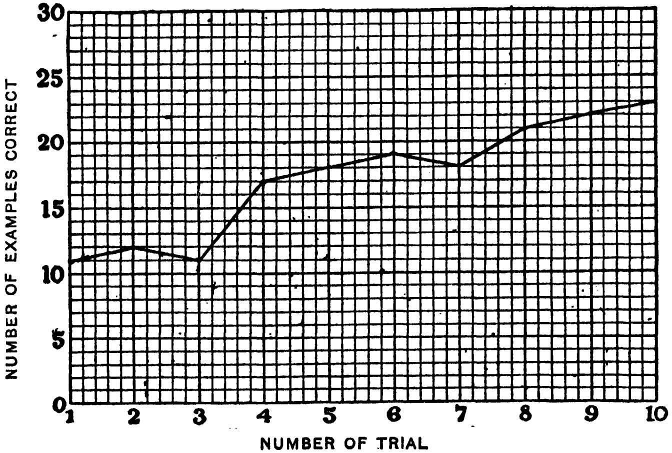
4. Show how you could make the graph from the table and how you could make the table if you had nothing but the graph to guide you.

5. Make a time-temperature graph from the facts given in the following table:

May .	1	2	3	4	5	6	7	8	9	10	11	12	13
Temperature (F.)	57°	56°	62°	66°	64°	65°	53°	66°	70°	69°	72°	69°	75°

6. Jack Smith made the following graph to show the

A GRAPH OF JACK'S IMPROVEMENT IN ADDITION



number of addition examples he did correctly in each of 10 trials.

7. Complete the table below, showing the same facts represented in the graph :

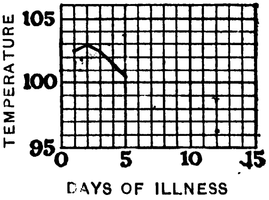
Number of trials .	1	2	3	4	5	6	7	8	9	10
Number correct .	11									

8. What does Jack's horizontal scale show ? What does the vertical scale show ?

9. How did Jack fix the points through which the graph is drawn ?

23. How to draw Line Graphs.—Mary Moore's nurse kept a graphic record of Mary's temperature. Part of the graph is shown here. On the sixth day her temperature was 102°. Show on the graph the point through which the temperature line must pass. On the seventh day her temperature was 103·5°. Show where this point would be on the graph. Copy this graph, and continue the temperature line through these two points.

GRAPHIC TYPHOID RECORD



The table here shows Mary's temperature during the remainder of her illness. Complete her temperature graph.

Day of illness .	1	2	3	4	5	6	7	8	9	10
Temperature (F.)	102·5	103	102·5	101·5	100·5	102	103·5	102	101·5	96·5

Points to remember in drawing line graphs:

(*a*) In making line graphs you need two scales—a horizontal and a vertical scale. It is very important to choose each scale wisely.

(*b*) Points must be fixed or plotted before the graph can be drawn.

(*c*) A point is plotted by means of a pair of numbers; one of these two numbers tells you how far to count on the horizontal scale, and the other number tells you how far to count on the vertical scale. For example, on the preceding graph the temperature on the sixth day was 102°. To plot the point through which the line must pass you had to count six units from zero to the right on the horizontal scale; then you had to go up far enough on the vertical scale to represent 102°.

(*d*) You need a table. A table is made up of pairs of numbers. Each pair is used to plot a point.

(*e*) The points obtained in (*c*) should be connected by straight lines for the kind of line graphs you are doing.

(*f*) The title of a graph should be clearly stated.

(*g*) Each scale should be labelled clearly to show what quantity is measured by it.

(*h*) Accuracy is essential and neatness is important.

EXERCISES II I

1. Philip Cox wished to improve in accuracy in mathematics. His school gave ten examinations for accuracy during the year. Philip's scores are given here:

Number of examination . . .	1	2	3	4	5	6	7	8	9	10
Per cent. of accuracy	60	65	80	75	70	70	80	85	85	90

Make a graph to show Philip's changes in accuracy. Measure per cent. of accuracy along the vertical scale.

2. Henry Blake had some doubt about the money value of a secondary school education. He found the following

report of a committee which had investigated the problem in the district in which he lived.

WEEKLY INCOME

—	Those who did not go to the Secondary School	Those who did go to the Secondary School
When 14 yrs. of age . .	4*s*.	—
,, 15 ,, ,, . .	7*s*.	—
,, 16 ,, ,, . .	10*s*.	—
,, 17 ,, ,, . .	15*s*.	—
,, 18 ,, ,, . .	£1	£1 5*s*.
,, 19 ,, ,, . .	£1 10*s*.	£2
,, 20 ,, ,, . .	£2	£2 15*s*.
,, 21 ,, ,, . .	£2	£3 15*s*.
,, 22 ,, ,, . .	£2 10*s*.	£4
,, 23 ,, ,, . .	£2 15*s*.	£4 5*s*.
,, 24 ,, ,, . .	£3	£4 15*s*.
,, 25 ,, ,, . .	£3 5*s*.	£5 5*s*.

(*a*) Draw a graph showing the increase in wages of a boy who leaves school at 14 years of age (*i.e.*, does not go to the secondary school).

(*b*) On the same piece of squared paper draw the wage graph of a boy who remains at school until 18 years of age (*i.e.*, does go to the secondary school).

(*c*) From a study of the two graphs which boy will probably advance more rapidly in later years ?

3. The average weight of boys for certain ages is given in the following table :

Age	6	7	8	9	10	11	12	13	14	15
Weight in lb. . .	50	53	57	62	67	72	78	85	93	105

Make a graph showing the increase in average weight of boys. On the horizontal scale use 1 unit to represent

1 year; on the vertical scale let 1 unit represent 10 lb. The zero of the vertical scale should be shown. Thus the scale should read 0, 10, 20, etc.

4. The table below shows the average weight of girls for the ages mentioned in question 3.

Age	6	7	8	9	10	11	12	13	14	15
Weight in lb. . .	43	47	52	57	62	69	78	89	98	106

Show graphically the increase in the average weight of girls. Sketch on the same piece of squared paper the curve you drew in question 3. This will enable you to compare the weights of boys and girls.

5. The best material for a graph will be that dealing with some problem in the life of your school. The preceding problems are suggested merely as practice material in case you fail to collect facts for a desirable graph. Here is a list of things which other pupils have pictured. It may suggest something to you: (*a*) personal expenditures during a month; (*b*) marks in different school subjects; (*c*) monthly sales in the school tuck-shop; (*d*) 'absences' or 'lates' in your form; (*e*) football matches, results or goals scored; (*f*) School Savings Bank deposits

24. Buying on the Instalment Plan.

Mr Buyer. We've been thinking of buying a piano. Would you sell us one like this on the *instalment* plan?

Mr Dealer. Oh, yes, we sell on the instalment plan. You pay a certain amount down and the rest in monthly payments. The price of this piano is 60 guineas. Would it be convenient for you to pay about 20 per cent. down and the rest in ten equal monthly instalments?

Mrs Buyer. Let's see. That would be —— cash, and —— a month for ten months.

Mr Buyer. What does one save by paying cash?

Mr Dealer. If you prefer to pay cash we give you 10 per cent. off, making the price ——.

Mr Buyer. I should think every one would want to pay cash; there is a saving of ——. But isn't your price on the instalment plan too high? I should think the profit would be very great.

Mr Dealer. The public doesn't seem to understand. I'll tell you, frankly, this piano which you buy for 60 guineas cost me 45 guineas. You see my profit is 15 guineas, or only —— per cent. You may consider that a high rate of profit. But suppose that the person who buys it should become unable to pay after the first instalment. I should have a second-hand piano in stock, and should have received only ——. I could never sell the used piano for more than 60 per cent. of the price of a new one, or ——. You see, I might actually lose as much as ——, plus all my expense.

Mrs Buyer. I'm convinced that it is better for you, and better for us, to do business on a cash basis. One of my neighbours doesn't seem to realize that she is really paying more when she buys on the instalment plan.

Mr Dealer. True, Mrs Buyer. But frequently the instalment plan is a great accommodation to people. My chief objection to it is that it sometimes encourages people to buy luxuries when they should be saving their money.

EXERCISES IIJ

1. A dealer paid £90 for a piano. He sold it for £125 on the instalment plan: 30 per cent. down and £10 a month. After four instalments had been paid he was obliged to take it back. Then he sold it for £35 cash. Did he gain or lose? How much? What per cent.?

2. Mr Henderson can buy a car for £190 cash, or for £100 cash and £10 a month for a year. Which is the better offer? What per cent. would he save by accepting the better offer? Do not consider interest on money.

3. The cash price of an incubator is £15. If bought on the instalment plan the price is £1 13*s*. 4*d*. down and £1 13*s*. 4*d*. a month for ten months. What per cent. would the customer save by choosing the better plan?

4. An encyclopædia is offered for £19 10*s*. down, or for

a first payment of £5 and 12 additional monthly payments of £1 10*s*. each. By how much per cent. is the amount paid on the instalment plan in excess of the single case payment ?

5. John Harker, Ltd., offers to furnish a flat for £145, £25 to be paid on delivery of the furniture and £10 a month to be paid for a year. A customer asks to be allowed to spread his payments over two years. To this the firm agrees on condition that he pays £5 5*s*. a month for two years. How much more per cent. does this customer pay than one who pays £10 a month for a year ?

6. A dealer paid £7 10*s*. for a bicycle. He sold it for £10 on the instalment plan : £4 down and 10*s*. a month for a year. After six instalments had been paid the buyer absconded, and the dealer got no further payments, but recovered possession of the bicycle, which he sold second hand for 30*s*. Did he gain or lose ? How much ?

7. A cuckoo clock is advertised for sale at 6*s*. 6*d*., but the advertisement states that 2*s*. 6*d*. down, followed by 3*d*. a week for 20 weeks, will be accepted. Which is the better offer ? What per cent. will you save by accepting the better offer ?

8. *The World's Greatest Dramas* comes out in twenty-four fortnightly parts, at 1*s*. 2*d*. a part. In the last part the publishers offer to bind the parts in two volumes, for 3*s*. a volume. A year later they republish all the parts in two volumes at half a guinea each. How much does one save by waiting ? How much per cent. ?

EXERCISES IIK

Questions on Voting

1. The girls of the Nightingale School organized a Girl Guide troop, with a membership of 42. They wanted to select a camping site Fifteen of the girls wished to choose

White's Hill. They met and voted to do so. The president of the club said that the meeting was unofficial. Why ?

2. The troop had agreed that at least two-thirds of the members, or —— girls, must be present before the meeting could do business. They said : "If two-thirds of our members are present we have a quorum." Could the troop conduct business when 27 girls were present ?

3. *The Oxford Dictionary* thus defines a quorum : "Fixed number of members that must be present to make proceedings of assembly or society or board valid." Explain the meaning of this definition. How many members were required to make a quorum in some meeting that you have attended ?

4. The girls voted on a proposed date for a carnival ; 21 voted for the date, and 15 voted against it. The leader said : "The majority of the members favour the proposed date." Majority meant *more than one-half of those voting*. How many votes are required for a majority when 30 votes are cast ? When 31 are cast ? When 19 are cast ? When 20 are cast ?

5. Majority often means *the number of votes more than all other candidates together received.* At a Guide election Grace received 22 votes for leader ; Nell received 11 ; and Helen 9. Grace was elected by a *majority* of —— votes.

6. John received 18 votes for form monitor ; Harry received 14, and Edward 12. John was elected by a *plurality* of 4 votes. What is the meaning of plurality ?

7. A plurality means *the number of votes more than the next highest candidate received.* How do you find John's plurality in question 6 ?

8. Did John receive a majority (see question 6) ?

9. Did you ever vote to decide which pupil in your class was the best all-round athlete ? Did any pupil receive a majority vote ? What was the good athlete's plurality ?

REVISION EXERCISES II

1. Boy Scouts and Army officers often measure distances by "pacing them off." An Army officer knows the length of his pace (stride), and uses it as a unit. Estimate the length of your ordinary pace. How could you determine the *average* length of your pace ?

2. Here is the method which a Boy Scout uses for determining the average length of his pace :

> (*a*) He paces off 100 ordinary strides, being careful to walk at a natural rate and being sure to mark the beginning and ending points of the distance.
>
> (*b*) He measures the total distance paced off.
>
> (*c*) He finds the average length of an ordinary stride by dividing the total distance by 100.

Why is this a good method ? Why does he get the *average* length of his stride ? Why just 100 strides ?

3. Determine the length of your ordinary pace.

4. A girl in business, living at home, spent 20 per cent. of her earnings on clothes, 12½ per cent. for music lessons, 10 per cent. for books, 17½ per cent. for a "general good time," gave her parents 15 per cent., and saved the remainder. Make a bar graph to show what she does with her earnings.

5. If the girl in question 4 earned in one year £65, how much did she spend for each item in one year ?

6. The boys in the Nelson School had a long-jump competition. The following were the jumps made : Sharpe, 8 ft. 6 in. ; Wilkins, 8·6 ft. ; Knight, 8 ft. 5 in. ; Aldridge, 8·25 ft. ; Halliday, 8·5 ft. ; Clarke, 8½ ft. ; and Morgan, 100 inches. Who was the best long jumper ? Who took second place ? Arrange the results in order.

7. Draw any two straight lines, *a* and *b*. Now draw a straight line $3a + 2b$ units long. Measure this straight line in inches. Compare the result with those obtained by others in your class. Did all pupils get the same result? If not, why not? On what does the result depend?

8. Draw, on squared paper, straight lines of the following lengths: 2·4 cm., 3·5 cm., 10 mm., 25 mm., 0·8 cm., 0·01 m., 2·45 cm., 3·75 cm., 0·1 cm., 3·95 cm.

9. Here are the distances from Liverpool to certain places, and the time it takes to make each journey:

Destination	Distance in Miles	Time in Days
Chicago	4,082	8
Mexico	5,447	14
Panama	4,576	19
Rio de Janeiro . . .	5,156	19
Vancouver	5,584	13
Yokohama	10,452	32
Rangoon	7,955	31

Draw two bar graphs, one to illustrate "Distance in Miles" in above table, and the other to illustrate "Time in Days" in above table. Work out the average number of miles per day for each journey, and then arrange the journeys in order of speed.

10. A group of pupils were comparing their marks in certain subjects. Which had the highest percentage mark?

(*a*) Parkinson had correct 3 out of 5 chemistry problems.
(*b*) Sutton spelt correctly 27 out of 30 words.
(*c*) Phillips had 15 questions in arithmetic right out of 18.
(*d*) Moore had only one out of 8 history questions wrong.
(*e*) Gardiner had 6 geography questions right out of 9.

Record the results for the above in a table like this :

Name	Part each one did, written as a Ratio	The Ratio expressed as a Decimal	Per Cent.
Parkinson . . .			
Sutton			
Phillips			
Moore			
Gardiner			

The complete table shows that —— had the highest marks, and that —— had the lowest marks.

11. A solution consists of alcohol and water in the ratio of 3 to 5. What part of the solution is water ? What part is alcohol ?

12. How much alcohol would there be in 40 gallons of the solution mentioned in question 11 ?

13. Draw a bar which contains three parts in the proportions 50 per cent., 40 per cent., and 10 per cent.

14. Draw a bar to show approximately an 80 per cent., 10 per cent., 10 per cent. division.

RUNNING THE ADDITION 'HURDLE' TEST

Many pupils can 'run' these hurdles in 10 minutes after five or six trials. More than one error knocks a hurdle down. All hurdles must be up when you finish. You should practise this test once a week until you run the hurdles.

Hurdle 1

(*a*) Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{4}$, 6, $1\frac{1}{4}$, $2\frac{1}{2}$, $\frac{3}{4}$, and 4.
(*b*) Add $8\frac{1}{5}$, $6\frac{1}{10}$, $9\frac{2}{5}$, $6\frac{7}{10}$, 8, and $4\frac{3}{10}$.
(*c*) Add $2\frac{1}{2}$, $5\frac{1}{4}$, $8\frac{3}{4}$, $9\frac{1}{2}$, $6\frac{3}{4}$, and 7.

Hurdle 2

(*a*) Add ·08, ·16, ·12, ·33, and ·62.
(*b*) Add ·2, ·8, 1·4, ·15, 2·02, and ·25.
(*c*) Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{6}$, $2\frac{1}{4}$, 6, and $\frac{5}{6}$.

Hurdle 3

(*a*) Add $\frac{3}{4}$, $2\frac{1}{3}$, $8\frac{1}{2}$, $6\frac{1}{4}$, $5\frac{1}{3}$, $4\frac{1}{6}$, and 12.
(*b*) Add $2\frac{1}{5}$, $4\frac{3}{10}$, $2\frac{3}{5}$, and $14\frac{7}{10}$.
(*c*) Add 2·1, ·21, 2·1, 21, 0·21, and 2·1.

Hurdle 4

(*a*) Add 8 ft. $5\frac{1}{2}$ in., 9 ft. $6\frac{1}{4}$ in., 3 ft. 5 in., and 7 ft. $8\frac{3}{4}$ in.
(*b*) Add 12 min. 40 sec., 8 min. 20 sec., and 6 min. 18 sec.
(*c*) Add 2 lb., 5 lb. 8 oz., 6 lb. 4 oz., 5 lb., and 4 lb. 12 oz.

PRACTICE FOR ACCURACY AND SPEED

Test 3. Division

8)32	9)81	4)32	1)0	8)72	2)18	8)64	2)0	8)24
9)36	7)63	3)21	3)0	6)54	6)24	7)49	4)0	3)24
5)25	6)48	9)27	5)0	5)45	4)28	7)42	9)0	8)56
5)30	8)40	4)32	6)0	6)36	7)35	5)35	8)0	6)18
7)56	9)72	3)18	7)0	5)20	9)63	7)21	9)54	4)36
4)24	8)48	3)27	9)45	7)28	6)42	6)30	5)40	9)9

Test 4. Adding by Endings

You should be able to add, mentally, any single-digit number to any two-digit number. For example, you should add 27 + 9 and instantly get 36 without thinking of carrying or counting. Just think: "7 + 9 = 16; 27 + 9 = 36."

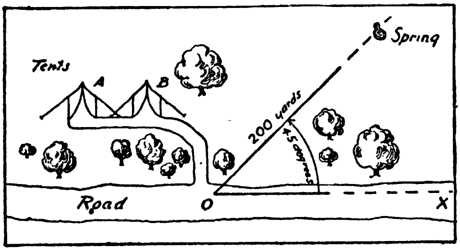
25 + 8
48 + 6
69 + 9
89 + 7
25 + 7
44 + 9
86 + 7
29 + 3
45 + 8
68 + 9
16 + 5
38 + 5
56 + 8
77 + 9
17 + 5
39 + 4
57 + 8
86 + 6
25 + 6
35 + 9
57 + 7
79 + 6
68 + 7
77 + 6
13 + 9
48 + 5
58 + 8
79 + 8
49 + 5
66 + 9
22 + 9
39 + 2
49 + 9
7 + 49
6 + 78
8 + 85
9 + 82
7 + 59
5 + 68
4 + 87
7 + 78
8 + 76
3 + 88
9 + 79

Remember to keep the daily record for each test. When you pass Test 4 turn to Test 5 on p. 86.

CHAPTER III

DIRECTION AND THE MEASUREMENT OF ANGLES

HOW TO FIND THE POSITION OF A PLACE



SKETCH OF CAMP

25. To find the Position of a Place we need both Distance and Direction.—Some Boy Scouts were planning a camping party. They sent three boys on ahead to make a sketch of the camping place. The sketch which the boys brought back to show at the next meeting is given here.

Exercises IIIa

1. Try to answer the following questions which were asked at the meeting of the Boy Scouts. Study the sketch above.

(*a*) What is the line marked with an O and an X ?

(*b*) How far is the spring from point O ?

(*c*) How would the scouts know in what direction to make the path from point O to the spring ?

(*d*) If you were at point O how would you tell the scoutmaster where the spring is ?

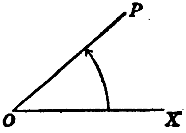
2. One boy said : " You get the direction by using the *angle.*" Another said : " We must make the path so that it shall make an angle of 45 degrees with the road." What did these boys mean ?

3. A third boy said : "All you need to know is that you walk 200 yards from the camp." Was he right ?

26. The Sketch of the Boy Scouts' Camp tells us both Direction and Distance.—That is, the line from the camp to the spring makes an angle of 45 degrees with the road, and the spring is 200 yards distant from the camp.

We often want to denote the position of a place. We do this by finding *how far* one place is from another place, and in what *direction.* Thus we say that our home is a mile north of the railway station. We have already learnt how to measure straight line distances. Now we need to learn how to measure direction.

27. We show Direction by Means of Angles.—In order to understand direction we need to study angles. Draw a straight line, say OX, as in this figure. Take your ruler, and place one end at O (place the edge of the ruler exactly along the line OX). Now keep the end fixed at O and turn the ruler about this point, but let it always lie on the paper. Stop turning the ruler anywhere you like, and draw a line along the edge in the new position. Call this line OP. The edge of the ruler has turned through an *angle.* The fixed



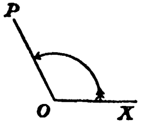
point O is called the *vertex*, and the lines OX and OP are called the *arms* of the angle.

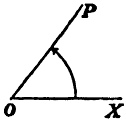
The angle is the *amount of turning*.

EXERCISES IIIB

1. What does the arrow show in the figure in § 27? Is it necessary?

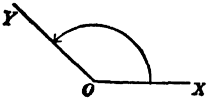
2. Draw the kind of angle you would have if the revolving line OP were turned about half-way round; about one-fourth of the way round; about three-fourths of the way round.

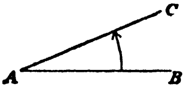




3. Look at the angles shown here. Decide which is the larger angle. Why?

4. Which of the angles shown below is the smaller? Why? Which has the longer arms?





5. Fill in the blank spaces in the following sentence: The size of an angle depends upon the amount ——, not at all upon the —— of the ——.

6. Imagine a clock-face with the minute hand at 12 (do not consider the hour hand). Where will the minute hand be when it has made one-fourth of a complete turn? When it has made one-half of a complete turn? When it has made two-thirds of a complete turn? When it has revolved three-fourths of the way round?

28. An Important Angle: the Right Angle.—Look at the right angle shown on p. 69. Point out a number of right angles in your classroom.

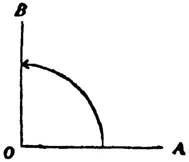
EXERCISES IIIc

1 What part of a complete revolution is a right angle.

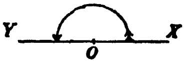
2. What amount of turning is represented by a straight angle? (See the figure.)

3. How many right angles are there in a straight angle?

4. Through how many right angles does the hour hand of a clock turn in 3 hours? In 6 hours? In 9 hours?

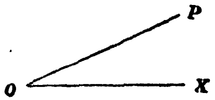


A RIGHT ANGLE



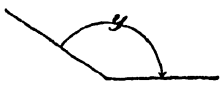
A STRAIGHT ANGLE

5. Is the angle XOP greater or less than a right angle? What name is given to an angle that is less than a right angle?



AN ACUTE ANGLE

6. What kind of angle is angle *y*? What is its relation to a right angle? To a straight angle? What name is given to an angle that is less than a straight angle and greater than a right angle?



AN OBTUSE ANGLE

7. Point to five acute angles in your classroom, and to three obtuse angles.

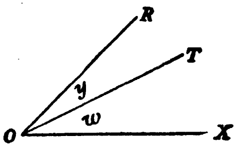
8. Find from a dictionary the difference in meaning between acute and obtuse.

9. What kind of an angle do the hands of a clock make at 2 o'clock? at 3 o'clock? at 6 o'clock? at 8 o'clock?

29. How to letter Angles so that they may be read easily.—There are three ways of lettering and reading angles. What these three ways are you will learn by doing the questions below.

Exercises IIId

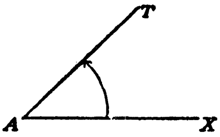
1. Place a freehand copy of this figure on the blackboard. Suppose that you wish to talk to your fellow pupils about some angle in this figure. Read the angle that you have in mind. Point to the angle. Now find out if your fellow pupils are all thinking about the same angle.



2. If you have been successful in question 1, try to read the angle in a different way and still be definite.

3. Point to angle *w*; to angle XOT. What do you notice about these two ways of reading angles ?

4. Place a freehand copy of the next figure on the board. Point to angle A; to angle XAT. When is it possible to read an angle merely by the use of the letter at the vertex ?



5. Tell in your own words the three ways that are used in reading angles.

6. Draw an angle. Letter it XOY. Remember that the middle letter is always placed at the vertex.

7. Draw an angle and letter it AOB. With the same vertex draw an angle AOC. What do the two angles have in common ?

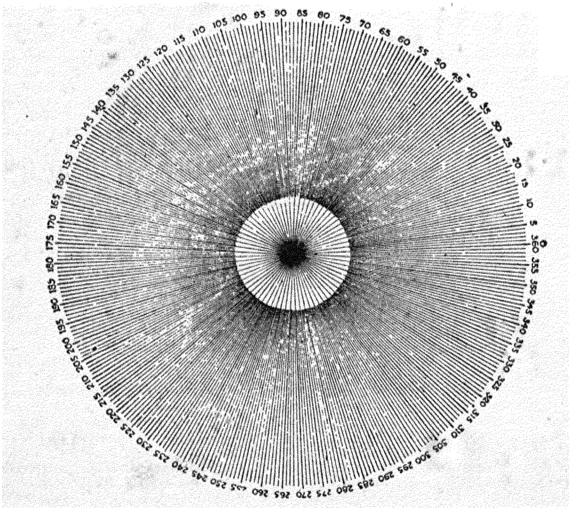
8. Draw two lines, AB and CD, cutting each other at O. Read four angles formed by the two lines.

30. The Unit with which we measure Angles is the Degree.—Just as we have units for measuring lines so we have units for measuring angles. The questions below will make clear what these units are.

EXERCISES IIIE

1. There are a great many small angles in this figure. The little angles are all equal. Each represents the same amount of turning as any other. How many little angles are there in the figure?

2. Each of these small angles is called a *degree.* How many degrees are there in a right angle?



ANGLES OF 1°

3. What part of a complete revolution is one degree? What part of a right angle is one degree?

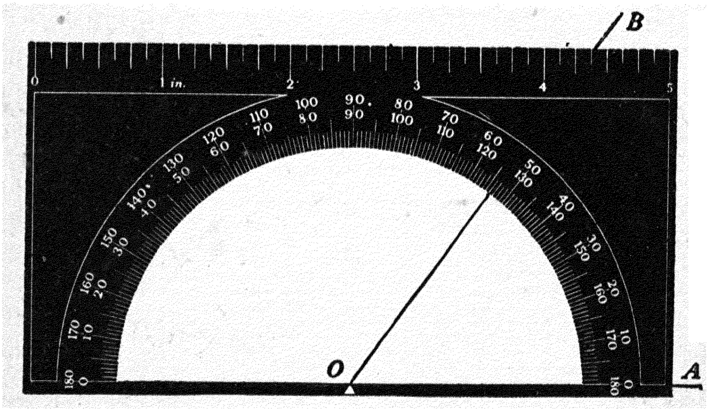
4. An angle of two degrees is often written as 2°. What is the symbol for degrees?

5. If a certain angle contains 30° it represents what part of a complete revolution?

6. If a complete revolution is marked off into 12 equal angles each angle will contain —— degrees.

7. Think of the hands of the clock when it is 3 o'clock. How many degrees are there in the angle formed by the hands? What angle is formed at 10 o'clock? At 1 o'clock?

31. The Protractor, an Instrument for measuring Angles.—The *protractor* is an instrument used for measuring turning or angles. The outside edge of the protractor is graduated (marked off) into degrees. The graduated edge is divided into 180 equal parts, called degrees.



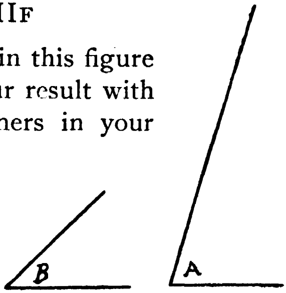
MEASURING AN ANGLE

In the figure above the protractor has been placed over the angle AOB to show you how to measure an angle. Note (*a*) where the centre of the protractor is placed, (*b*) where the line OA is, (*c*) where the line OB is.

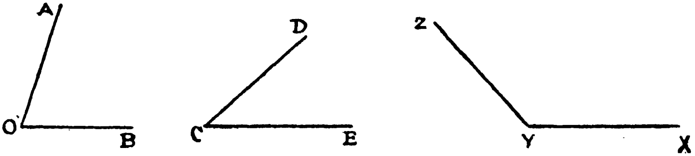
EXERCISES IIIF

1. Measure each of the angles in this figure with a protractor. Compare your result with the results obtained by the others in your class.

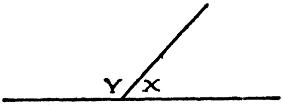
2. What difficulty did you have in measuring angle B that you did not have in measuring angle A ?



3. Measure with your protractor these angles :



4. Measure the angle X in this figure. Show how to find the angle Y without measuring it.



5. (*a*) In the figure above why does $X + Y = 180^\circ$?
(*b*) If $X = 50^\circ$, $Y =$ ——.
(*c*) If $X = 60^\circ$, $Y =$ ——.

6. How many degrees are there between the hands of a clock at 2 o'clock ? At 4 o'clock ? At 6 o'clock ? At 9 o'clock ?

7. Put away your protractor and draw, by estimate, an angle of 60°. Now measure with the protractor the angle you have drawn. What was the amount of your error in estimating 60° ?

8. Draw, by estimate, an angle of 45° ; of 80° ; of 135° ; of 30° ; of 75°.

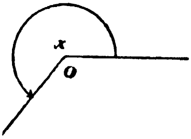
9. Measure each of the angles you drew for question 8

and make a record of the results in a table like the one shown here :

Estimated . . .	45°	80°	135°	30°	75°
Measured result . .					
Error					

10. Consider an error of 5° or less satisfactory. How many of your results in the above table are satisfactory ?

11. If a pupil has one satisfactory result, what per cent. efficient is he ? How efficient are you in estimating the size of an angle ?



12. Measure the angle X in this figure. What difficulty is there in doing this ?

13. How large is the angle O in the above figure ? Explain how you could find the number of degrees in this angle without measuring it if you knew the number of degrees in angle X.

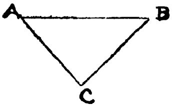
14. Draw two straight lines that intersect (cross) each other. Measure any one of the angles formed. Now measure the angle that is *opposite* to the one you measured. What conclusion seems to be true ?

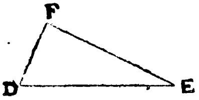
32. Measuring the Angles of a Triangle.—There is a very important fact about the angles of a triangle. What this is you will find out as you work through the next set of exercises.

Exercises IIIg

1. Draw any triangle ABC and measure its angles. Draw several triangles and measure their angles.

2. (*a*) Measure the angles A, B, and C of the triangle ABC, and record your results in a table like the one below.





(*b*) Measure the angles D, E, and F in the triangle above, and record your results in the table.

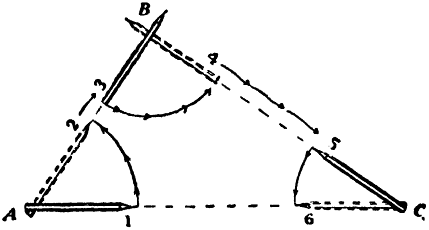
Angle	Triangle ABC	Triangle DEF	Your Triangle
First . . .			
Second . .			
Third . .			
Sum . . .			

3. Find the sum of the three angles for each triangle, and record it in the proper place.

4. Now draw any triangle. Measure each angle, and enter your results in the table.

5. About what number do the sums in the above table seem to centre ?

6. Draw a triangle as suggested by this figure. Note the direction in which the pencil points in position 1. Rotate the pencil through angle A, as shown by the arrows. Then slide it along AB to the position indicated in the figure. Rotate the pencil next through angle B, as indicated, and slide it along BC to the position shown. Then rotate the pencil through angle C to the last



position shown. In what direction does the pencil now point? What part of a complete turn has it made? Through how many right angles has it turned? Through how many degrees has it turned? How does the result of this experiment support your conclusion in question 5?

7. If two angles of a triangle are 30° and 70°, how large is the remaining angle?

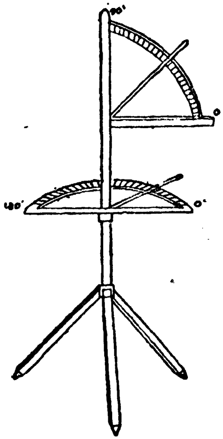
8. Find the third angle for triangles in which the first and second angles are 50° and 80°; 20° and 100°; 170° and 5°; 72° and 28°; 1° and 49°; 160° and 19°.

9. If one angle of a triangle is 90° what is the sum of the remaining two? What kind of angle would each be?

10. How many right angles can a triangle have? How many obtuse angles? How many acute angles?

11. In the equation $x+y+z=180°$, if y is 50° and x is 80°, how large is z?

12. What is x in the equation $x+y+z=180°$, if y is 50° and z is 70°?



13. If two angles of a triangle are each 50°, how large is the third angle?

14. If the three angles of a triangle are all the same size, how large is each?

15. One angle of a triangle is 90° and another is 30°. How large is the third angle? How large would the third angle be if the first were 90° and the second were 45°?

33. The Field Protractor.—An instrument, called a *field protractor*, which will measure angles approximately, can easily be made. (See the figure.) It consists of two protractors, one horizontal and one vertical, fastened to a rod on a tripod. You may wish to make a field protractor and measure angles.

34. The Divisions of the Degree.—The degree is divided into 60 equal parts, each of which is called a *minute*. The symbol for minute is ′. Again, the minute is divided into 60 equal parts, each of which is called a *second*. The symbol for second is ″.

EXERCISES IIIH

1. What fractional part of a minute is a second? What part of a degree would such a very small angle be?

2. How many minutes are there in 90°?

3. How many minutes are there in 40° 20′?

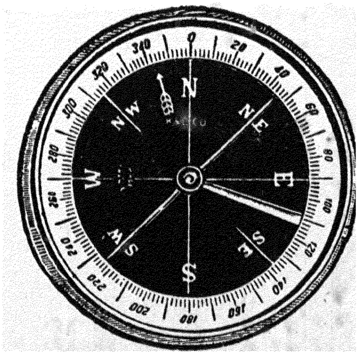
4. How many seconds are there in one complete revolution?

5. Find the number of seconds in each of these angles:

(*a*) 10° 12′ 20″
(*b*) 45° 25′ 30″
(*c*) 60° 50′ 10″
(*d*) 37° 23′ 19″

6. Explorers and sailors use an instrument called a compass with which to tell direction. Here is a picture of one. How does this resemble a double protractor?

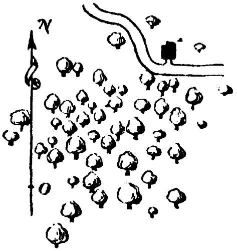
7. What additional parts does it have that a protractor does not have?



A COMPASS

8. In the next figure O represents a school on a road

running north and south. P represents Brown's home beyond the woods and exactly north-east of the school. Show how Brown might walk home from school in a straight line by the aid of a compass.



9. George started at a point P; he walked north-west 1 mile, then due south 2 miles, and then south-east 1 mile. How far was he then from P? Sketch his path.

35. Heights and Weights.—The table below shows

Height and Weight Table

	For Girls				For Boys			
Height in Inches	12 Years	13 Years	14 Years	15 Years	12 Years	13 Years	14 Years	15 Years
50	61	—	—	—	62	—	—	—
51	64	—	—	—	65	—	—	—
52	67	—	—	—	68	—	—	—
53	69	70	—	—	70	71	—	—
54	72	73	—	—	73	74	—	—
55	75	76	77	—	76	77	78	—
56	79	80	81	—	80	81	82	—
57	83	84	85	86	83	84	85	86
58	87	88	89	90	86	87	88	90
59	91	93	94	95	89	90	92	94
60	95	97	99	100	93	94	97	99
61	101	102	104	106	97	99	102	104
62	106	107	109	111	102	104	106	109
63	111	112	113	115	107	109	111	114
64	115	117	118	119	113	115	117	118
65	117	119	120	122	—	120	122	123
66	119	121	122	124	—	125	126	127
67	—	124	126	127	—	130	131	132
68	—	126	128	130	—	134	135	136

you the normal weight of boys and girls according to their age and height.

This table should be read as follows: A 12-year-old girl who is 55 inches in height should weigh 75 pounds; a 12-year-old boy who is 55 inches in height should weigh 76 pounds.

EXERCISES IIIi

On the Use of the Height and Weight Table

1. Jane Brown is 14 years old, weighs 102 pounds, and is 64 inches high. How much above or below the standard is her weight?

2. What is the standard weight for a 13-year-old boy who is 58 inches high? 66 inches high? 54 inches high? For a 12-year-old girl who is 5 feet tall?

3. Jack Henderson, a 12-year-old schoolboy, is 52 inches tall and weighs 64 pounds He is —— pounds too —— for his age.

4. Grace Snaith, who is 13 years old, 53 inches tall, and weighs 73 pounds, weighs —— pounds too —— for her age.

5. Compare your own weight with the standard weight for a person of your age and height.

6. What per cent. of the members of your class are of normal or standard weight? Below normal? Above normal?

7. Four pupils in a certain school were below standard weight for their age and height. They went on a careful diet, and kept the following record of their weights (see table on next page).

Name	Age	Standard Weight	Actual Weight					
			Nov. 1	Dec. 1	Jan. 1	Feb. 1	Mar. 1	Apr. 1
Collins (boy) .	11	88	$81\frac{1}{8}$	$84\frac{3}{4}$	$82\frac{1}{2}$	84	$85\frac{7}{8}$	82
Dixon (boy) .	11	85	81	$83\frac{1}{2}$	85	$85\frac{1}{4}$	85	86
Davis (girl) .	12	75	67	68	$69\frac{3}{8}$	$70\frac{1}{2}$	71	71
Burton (girl) .	9	69	$64\frac{1}{2}$	$63\frac{1}{2}$	$64\frac{5}{8}$	66	67	$67\frac{3}{4}$

To find out whether they were making satisfactory gains they secured a table which told them the amount that boys and girls should gain each month. The table is shown below. (*a*) Does the weight of a person increase *regularly*? (*b*) In which months did Collins gain the right amount? What was his average gain during the five months? (*c*) Find Dixon's average monthly gain. (*d*) Did any pupil fail to make the average gain that a pupil of his age is supposed to make?

ABOUT WHAT A BOY OR GIRL SHOULD GAIN EACH MONTH

Age	Girls	Boys
	oz.	oz.
5 to 6 . . .	5	5
7 to 8 . . .	7	7
9 to 10 . . .	9	9
11 to 12 . . .	11	10
13 to 14 . . .	14	12
15 to 16 . . .	12	14
17 to 18 . . .	6	8

8. The preceding table furnishes us facts for an interesting line graph. Use the horizontal axis to represent years

and the vertical axis to show gain in ounces. Using the figures given for girls, plot points as you did in making temperature line graphs. Draw a smooth curve through the seven points. Now use the line graph to answer such questions as : How much does a girl gain from 14 to 15 years ? From 8 to 9 years ? When is the gain most rapid ? Note that some of these questions cannot readily be answered from the above table. Take the figures given for boys, and draw a line graph on the same set of axes. Compare the two graphs.

9. Fred Halliday was 13 on June 1. In June his weight increased by 9 oz., in July by 7 oz., in August by 8 oz., in September by 10 oz., in October by 13 oz., in November by 15 oz. What was his total increase in weight for the six months ? How much was this below the average ? How much per cent. below the average ? What should be his average monthly gain in weight during the next six months, if his gain in weight for the year is to be normal ?

36. Using Business Methods in Spending. The Budget System.—To most people the word ‘ budget ’ suggests the national Budget which the Chancellor of the Exchequer introduces in the House of Commons every April. This national Budget is an estimate of the national income and the national expenditure for one year. The national income is derived from taxes, and if the national expenditure is increasing new taxes will have to be made or old ones increased. But every household may have a budget — indeed, must have a budget if money affairs are to be conducted on businesslike methods.

Let us consider two typical budgets: the first that of a working man who is a bachelor and lives in rooms; the second that of a family.

TABLE 1.—WEEKLY BUDGET OF A WORKING MAN (A BACHELOR)

Income	£	*s.*	*d.*	*Expenditure*	£	*s.*	*d.*
Wages	3	5	0	Board and lodgings	1	10	0
				Fares (6 days)	0	3	0
				Meals (not at lodgings)	0	7	6
				Papers, tobacco, and cigarettes	0	5	6
				Amusements	0	2	0
				Insurances and clubs	0	2	6
				Odd expenses	0	2	6
				Saved	0	12	0
	£3	5	0		£3	5	0

TABLE 2.—WEEKLY BUDGET OF FOUR FAMILIES (INCOME £5 A WEEK)

	2 Persons			3 Persons			4 Persons			5 Persons		
	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
Rent and rates	0	8	0	0	8	0	0	10	0	0	10	0
Housekeeping (including food)	1	16	0	2	0	0	2	8	0	2	16	0
Fuel and light	0	5	0	0	5	0	0	6	0	0	6	0
Clothes	0	7	6	0	7	6	0	9	0	0	10	0
Working expenses (fares, insurance, etc.)	0	3	6	0	3	6	0	5	3	0	7	0
Total for necessaries	£3	0	0	£3	4	0	£3	18	3	£4	9	0
Amusements, etc., and savings	2	0	0	1	16	0	1	1	9	0	11	0
Total	£5	0	0	£5	0	0	£5	0	0	£5	0	0

EXERCISES IIIK

1. Draw out a weekly budget for a family of 3 (father, mother, and one child), the weekly income being £4 10*s*.

2. Draw out a weekly budget for a family of 5 (father, mother, and 3 children), the weekly income being £5 10*s*.

3. Draw out a weekly budget for a household of 4 (father and mother, too old to work, their son, and their son's wife), the weekly income being £3 15*s*.

4. In Table 1 above what per cent. of the income is saved? What would you suggest for increasing the amount saved?

5. In Table 2 above find for each of the four families what per cent. of income is available for "Amusements, etc., and Savings"?

6. In Table 2 above find for each of the four families what per cent. of income is spent on housekeeping?

7. A man finds he spends on clothes £19 10*s*. a year. How much must he set aside for clothes in his weekly budget?

8. A man's quarterly gas bills during one year were £2 17*s*. 6*d*., £2 2*s*. 6*d*., £1 15*s*., and £3. During the same year he burnt $3\frac{1}{2}$ tons of coal at 45*s*. 6*d*. a ton. How much should he set aside for fuel and light in his weekly budget through the following year?

37. The More you do the More you earn.—A certain company engaged in the manufacture of boxes decided to encourage its employees to work rapidly and efficiently by paying them as follows: (*a*) for every perfect box up to 50 a day the company pays at the rate of 1*d*. each; (*b*) for every additional box per day up to 75 it pays 3*d*.; (*c*) for every box over and above 75 per day, it pays 4*d.*; (*d*) if a worker turns out an imperfect box she receives no pay for it; on the contrary, she forfeits for every spoiled box 1½*d.* of her pay to cover part cost of the material she has wasted.

EXERCISES IIIL

1. From the information given above reckon the total weekly earnings of Minnie Jones, one of the employees, whose record was as follows:

Monday:	56	perfect,	5	spoiled
Tuesday:	64	"	0	"
Wednesday:	78	"	1	"
Thursday:	82	"	3	"
Friday:	76	"	0	"
Saturday:	40	"	4	"

2. Gertrude Wheeler held the record for speed and efficiency: 84 boxes perfect and no boxes spoiled in 8 hours. She earned —— for the day's work, or —— pence an hour.

3. Sarah Brown's record during the first week of her employment by the company was as follows:

Monday:	32	perfect,	13	spoiled
Tuesday:	43	"	11	"
Wednesday:	51	"	8	"
Thursday:	55	"	7	"
Friday:	59	"	4	"
Saturday:	40	"	4	"

Find her average daily earnings.

REVISION EXERCISES III

1. Write a sentence containing these words: *line*, *fixed point*, *a plane*, *amount of turning*, and *an angle is*.

2. Arrange in order of size: obtuse angle, right angle, angle of 60°, straight angle, and angle of 30°

3. One way to denote the position of a place is to give its —— and its —— from some fixed place.

4. The direction a line takes is measured by the —— it makes with some reference line.

5. Some of the following statements are true; others are false. Make a list of those that are true.

(*a*) If we make the sides of an angle longer we make the angle larger.

(*b*) A degree is $\frac{1}{360}$ of a complete revolution.

(*c*) The sum of the angles of a triangle is equal to two right angles.

(*d*) If we measure a certain angle with a large protractor we get a greater result than if we measure it with a smaller protractor.

(*e*) A surveyor measures angles with a field protractor.

(*f*) All right angles are equal.

6. Mr Thomas was anxious to have his cherries picked as quickly as possible. Two boys of the neighbourhood, Jack and Frank Turner, agreed to pick them on this plan: that they should receive 4*d.* a gallon for the first 20 gallons picked in a day, 6*d.* a gallon for the next 10 gallons picked in a day, and 9*d.* a gallon for each gallon over 30 gallons a day. The boys picked 28 gallons the first day and 33 gallons the second day. How much did they receive for their services?

7. Dick Barton kept a record of the improvement he made in the long jump. His records were: February, 7 ft. 4 in.; March, 7 ft. 10 in.; April, 8 ft. 3½ in.; May, 8 ft. 7 in.; June, 8 ft. 9 in. Represent his improvement graphically.

8. Dressmakers often measure cloth by stretching it from the tip of the nose to the end of the arm. Stretch a piece of string from the tip of your nose to the end of your

outstretched arm ; then measure this distance. Can you reach a yard in this way ? What do you think of the accuracy of this method of measuring ?

9. On a map in a certain atlas two towns are $2\frac{7}{8}$ inches apart. What is the actual distance between the towns if $\frac{1}{4}$ inch on the map represents 25 miles ?

10. How many pounds in £$(2x+y)$ if $x=$£4 10*s*. and $y=$£7 ?

11. It is impossible to get very accurate measurements of length without using a —— unit of measure.

12. Harry paid 6*s*. per hundred for newspapers, and sold them at 1*d*. each. How much did he make if he sold 175 papers a day ?

13. At a certain school picnic the school treated each child to ice-cream. What facts did they need to know to purchase the right amount of ice-cream ?

PRACTICE FOR ACCURACY AND SPEED

Test 5. Practice in Multiplication

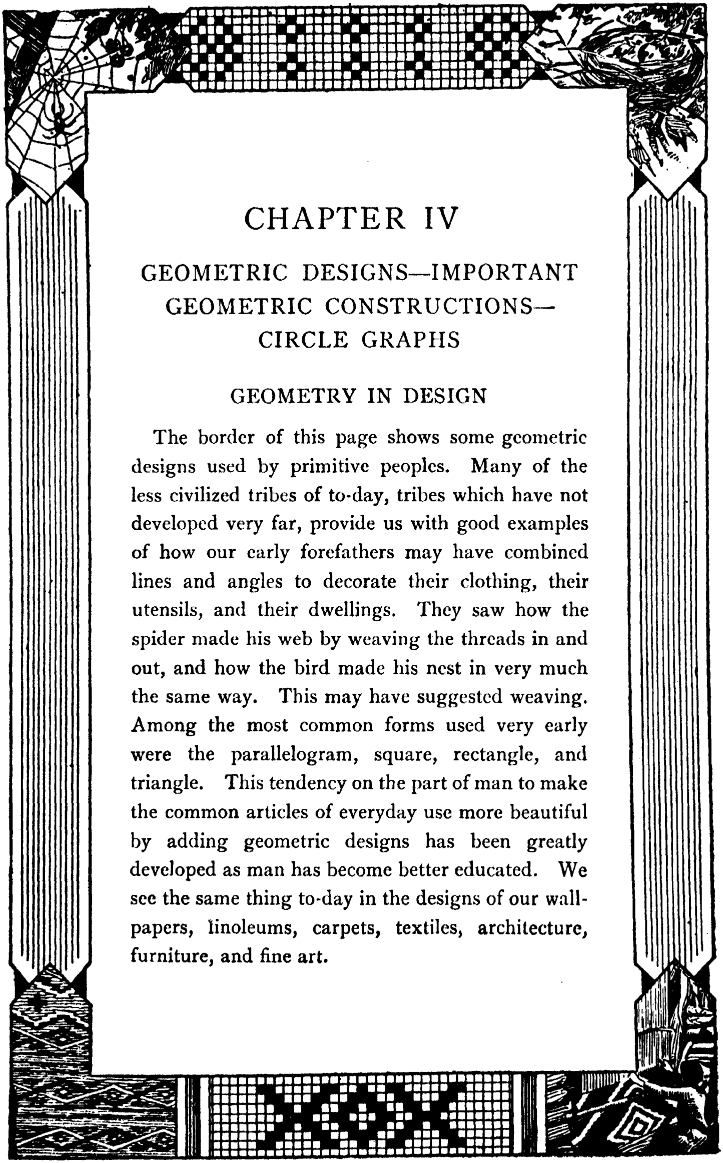
630	840	720	950	820	806	840
45	40	39	30	67	59	72
509	601	809	705	403	170	107
51	35	45	63	57	90	21
904	608	509	307	809	608	809
60	20	50	80	40	70	11

Test 6. Measuring my Skill in Subtraction

166,975 85,432	179,071 89,561	72,240 30,476	148,128 81,830
103,867 33,117	138,714 60,938	82,832 27,316	211,563 66,691
94,136 26,208	130,593 97,997	80,818 32,420	87,180 37,089
19,104 3,632	13,418 8,736	131,661 82,725	897,619 258,257

If you have passed six tests during the first six weeks at school you have reason to feel encouraged. Each test that you pass gives you added power. It is likely that you will pass later tests with less practice. Remember to keep careful records of your progress.

When you pass Test 6 turn to Test 7, p. 114.

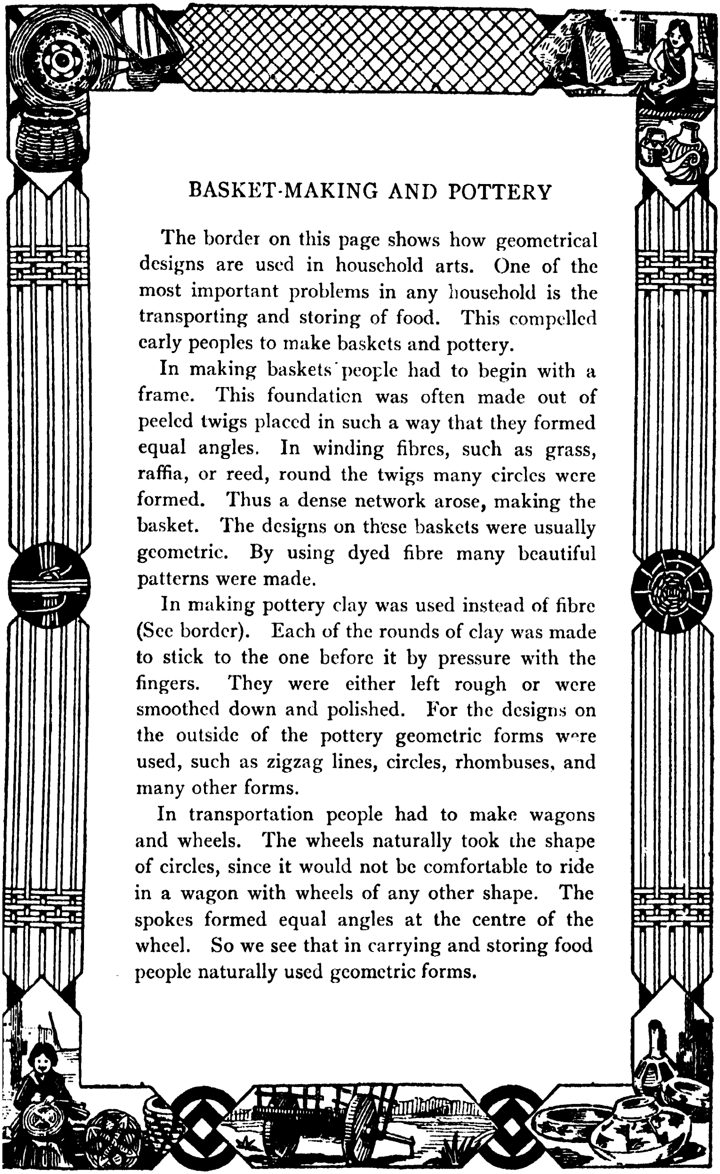


CHAPTER IV

GEOMETRIC DESIGNS—IMPORTANT GEOMETRIC CONSTRUCTIONS—CIRCLE GRAPHS

GEOMETRY IN DESIGN

The border of this page shows some geometric designs used by primitive peoples. Many of the less civilized tribes of to-day, tribes which have not developed very far, provide us with good examples of how our early forefathers may have combined lines and angles to decorate their clothing, their utensils, and their dwellings. They saw how the spider made his web by weaving the threads in and out, and how the bird made his nest in very much the same way. This may have suggested weaving. Among the most common forms used very early were the parallelogram, square, rectangle, and triangle. This tendency on the part of man to make the common articles of everyday use more beautiful by adding geometric designs has been greatly developed as man has become better educated. We see the same thing to-day in the designs of our wall-papers, linoleums, carpets, textiles, architecture, furniture, and fine art.



BASKET-MAKING AND POTTERY

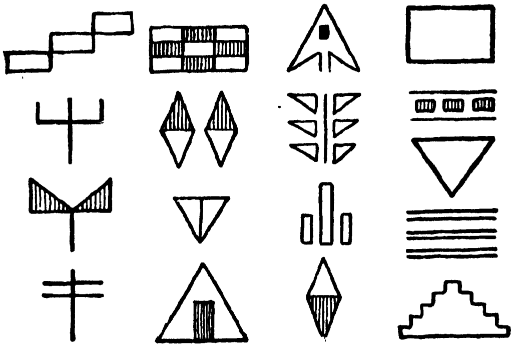
The border on this page shows how geometrical designs are used in household arts. One of the most important problems in any household is the transporting and storing of food. This compelled early peoples to make baskets and pottery.

In making baskets people had to begin with a frame. This foundation was often made out of peeled twigs placed in such a way that they formed equal angles. In winding fibres, such as grass, raffia, or reed, round the twigs many circles were formed. Thus a dense network arose, making the basket. The designs on these baskets were usually geometric. By using dyed fibre many beautiful patterns were made.

In making pottery clay was used instead of fibre (See border). Each of the rounds of clay was made to stick to the one before it by pressure with the fingers. They were either left rough or were smoothed down and polished. For the designs on the outside of the pottery geometric forms were used, such as zigzag lines, circles, rhombuses, and many other forms.

In transportation people had to make wagons and wheels. The wheels naturally took the shape of circles, since it would not be comfortable to ride in a wagon with wheels of any other shape. The spokes formed equal angles at the centre of the wheel. So we see that in carrying and storing food people naturally used geometric forms.

38. Geometric Designs.—The geometric forms that you see here are taken from the border on pp. 88, 89. You will see in shops, in buildings, in furniture, and in art many figures which are formed by lines and angles.



Exercises IVa

1. Count the number of triangles in the above simple geometrical forms.

2. How many right-angled triangles (triangles having one angle of 90°) do you see? How many isosceles triangles (two sides equal) can you find?

3. Say which of the following figures you find: the square, parallelogram, rectangle. Which figures contain parallel lines?

4. Which of the forms above do you think would make a good simple repeating border design?

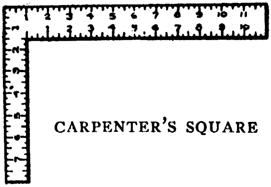
5. See if you can make any other geometrical forms similar to the above. Aim at making them simple and striking.

39. Different Ways of drawing the Right Angle.—Since straight-line figures are used so often in our furniture, our clothing, our buildings, and the like, it is necessary to know how to construct these figures accurately. To draw the designs in the preceding figure one must know how to use instruments for the construction of angles of different kinds, perpendicular lines, and parallel lines. The first and most important angle to know how to construct is the right angle. When two lines form a right angle each is said to be *perpendicular* to the other. The symbol for perpendicular is $\perp$. There are several ways of constructing a right angle.

Exercises IVb

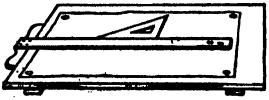
1. Draw a right angle, using a ruler and a protractor.

2. This is a picture of a carpenter's square, an instrument with which a carpenter rules guide lines on boards so that he can saw straight. Show how this instrument can be used in the construction of a right angle.



CARPENTER'S SQUARE

3. These tools are used by draughtsmen to draw angles and parallel lines. Show how a right angle can easily be drawn by using these tools.



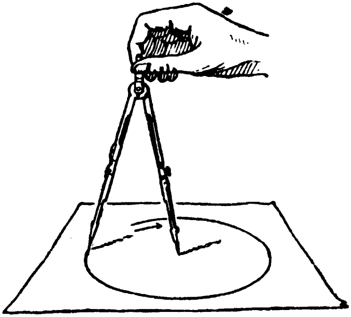
T-SQUARE AND SET-SQUARE

40. Circles: how to draw and use them.—Circles are usually drawn with the compasses. One point of the compasses is set at the *centre* of the circle.

The line that is traced by the moving point of the compasses is the *circumference* of the circle.

EXERCISES IVC

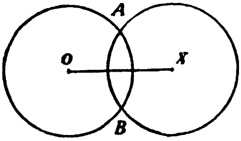
1. Open the compasses so that the distance between the points is 1 inch. Then draw a circle, keeping this opening fixed.



2. What is the distance from the centre of the circle to the circumference in question 1? This distance is called the *radius* of the circle. Draw a circle with a $\frac{3}{4}$-inch radius.

3. What is the longest line that can be drawn within a circle? This line drawn through the centre is called the *diameter*. Draw a circle with a diameter of $1\frac{1}{2}$ inches.

4. An arc of a circle is a part of the circumference of a circle. Draw an arc of a circle which has a 2-inch radius.



5. How would you make an exact copy of this figure? Where would you place the centres of the circles? Which would you draw first, the line OX or the two circles? Why? Make a copy and join points A and B. How large is the angle formed by AB and OX? Test your answer with a protractor. Is AB perpendicular to the horizontal line? Could you draw perpendicular lines by using two circles? How?

41. How to draw Right Angles with the Compasses.—Now that we have studied circles we are ready to use them in drawing a right angle. Study the following directions until you are quite sure you know how to draw a right angle with the compasses.

(1) Lay down a line MN (Fig. 1). Mark a point, P, at which, we will say, you want to erect a perpendicular to MN.

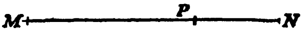


FIG. 1

(2) With P as centre and a convenient short radius mark off two equal parts, PD and PE, on either side of P (Fig. 2).

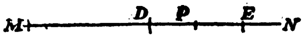


FIG. 2

(3) Now with D and E as centres and radius a little more than PD or PE, draw two arcs intersecting at some point, say R (Fig. 3).

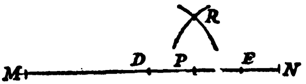


FIG. 3

(4) Draw a straight line from R to P. The construction is now complete. The angle RPE is a right angle —*i.e.*, PR is perpendicular to MN.

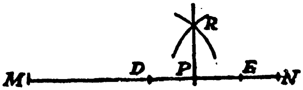


FIG. 4

EXERCISES IVD

1. How did you get the points D and E in (2) above?
2. How did you find the point R above?

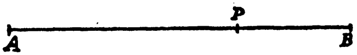


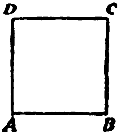
FIG. 5

3. Copy Fig. 5, and draw a line perpendicular to AB at the point P.
4. Draw a line AB on the blackboard. Mark a point in the line and call it R. At the point R construct a line perpendicular to AB. (In doing this exercise use a string tied to a piece of chalk instead of the compasses.)

42. How to construct Squares and Rectangles.— We learn to draw right angles so that we can draw correctly such important figures as squares and rectangles. Probably no regular figures are more commonly used in our everyday life than these right-angled figures.

Exercises IVe

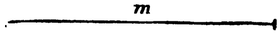
1. How many angles has a square? What kind of angles?



2. How many sides has a square? What relation is there between the sides?

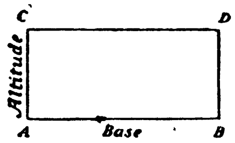
3. With a protractor and a ruler draw a square, each of whose sides measures 2 in.

4. With compasses and ruler (do not use the scale) draw a square whose sides are equal to the line m shown here.



5. How does a rectangle differ from a square?

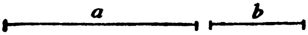
6. If you were told to draw a particular rectangle, of how many lines would you need to know the length? Of how many lines would you need to know the length in order to draw a particular square?



7. The line AB is called the *base* of the rectangle, and AC the *altitude*.

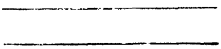
With ruler and protractor draw a rectangle whose base is 3 in. and whose altitude is 2 in.

8. With compasses and ruler draw a rectangle, using line b shown here as the base and a as the altitude.



9. Show that angles A + B + C + D = 360° in the rectangle drawn for question 7.

43. The Meaning of Parallel Lines.—You see here a pair of *parallel lines*. We shall now learn what parallel lines are and how to draw them.

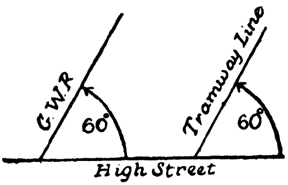


EXERCISES IVF

1. What angle does the railway line make with the High Street in this figure? What angle does the tramway line make with the High Street?

2. Do the railway line and the tramway line run in the same direction?

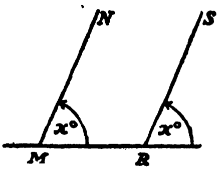
3. If they continue in their present direction will the railway line and the tramway line ever meet?



4. Is the G.W.R. line parallel to the High Street? What lines in the figure are parallel? Why?

5. Draw the line AC as shown here. With a protractor draw through point A a line making an angle of 50° with the line AC. Call this line AD. Now draw through point C a line making an angle of 50° with the line AC, produced. Call this line CB. We say that two lines have the same direction when they have had the same amount of revolution from some fixed line. Show that AD and CB have the same direction. This means, of course, that you are to show that AD and CB are parallel.

6. Show that MN and RS are parallel. (Note that the two angles are equal.)

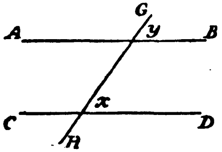


7. Point to four examples of parallel lines in your classroom.

8. The symbol for "is parallel to" is ||. Using the symbol, write "AB is parallel to CD" on the blackboard. Read the following: MN || RS; XY || CD; XY ⊥ CD.

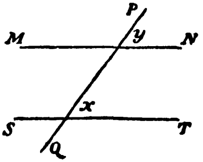
9. (*a*) Draw a line XY to represent a north-south road. Mark two points, A and B, on this line to represent the positions of two farmhouses. Now draw two lines to represent two roads which run north-east, one from each of the houses. Show that these roads are parallel.

(*b*) Draw a line on the blackboard. Mark two points, P and Q, on this line. Take two pointers or rulers and rotate them about these points so that the rulers are always parallel.



(*c*) What have angles to do with keeping the two lines parallel?

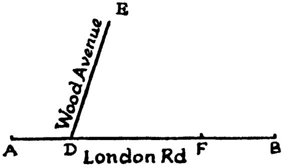
10. The angles x and y in this figure are called *corresponding* angles. Show that this is a good name for these angles. What will have to be true of x and y to make AB || CD?



11. A pupil drew the lines MN and ST parallel. Is it necessary to measure angles x and y to know that they are equal? Why?

12. Draw two lines AB and CD, and a crossing line GH, as in question 10. See how many pairs of corresponding angles you can find in the figure.

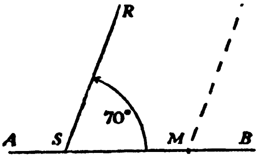
44. How to draw a Line parallel to another Line.—In the figure AB represents a road, the London Road; DE, Wood Avenue, is a road off the London Road. Suppose at the point F it

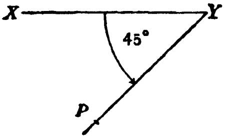


is required to make a road parallel to Wood Avenue. Our problem here is: How to draw through a given point a line parallel to a given line. You will learn how to do this in the exercises that follow.

EXERCISES IVG

1. Suppose that in the following figure we wish to draw a line through the point M so that it will be parallel to RS.





What angle will it have to make with AB?

2. Now suppose that we wish to draw a line through the point P so that it will be parallel to XY. (See the figure at the right.) What angle will the line have to make with PY?

3. A master gave his class this problem: "Make a copy of Fig. 1; then draw a line through the point P so that it will be parallel to the line AB."

The best answer that he received from his class was the following:

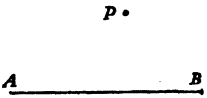


FIG. 1

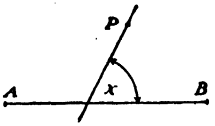


FIG. 2

(*a*) First, draw any line through P and cutting AB, as in Fig. 2.

(*b*) With a protractor measure the angle that this line forms with AB.

(*c*) Now with P as a vertex, and with this helping line as one side, make an angle equal to this angle, as in Fig. 3.

(*d*) The other side of this new angle will be parallel to AB. (See Fig. 3.)

Show at the blackboard how the pupil did the problem. Have you any suggestions for improving either the method or the explanation?

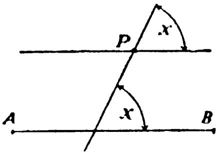


FIG. 3

4. Copy Fig. 4, and draw through the point P a line parallel to AB. Follow the steps given in question 3.

5. Is XY parallel to CD in Fig. 5? Do you need to use a protractor to determine whether or not XY is parallel to CD?

6. Show that the opposite sides of a square are parallel. Show that the same is true of a rectangle.

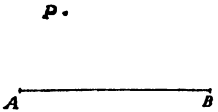


FIG. 4

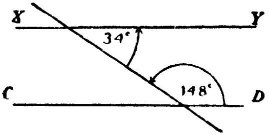


FIG. 5

7. Write a sentence in which you use the phrases *parallel lines*, *same direction*, *a fixed line*.

8. Write a sentence in which you use these phrases: *parallel lines*, *a crossing line*, *equal corresponding angles*.

45. How to draw Parallelograms.—*Parallelograms* are of common occurrence in design and construction work. A parallelogram is a four-sided figure whose opposite sides are parallel. Show that you know how to construct a parallelogram with compasses and ruler.

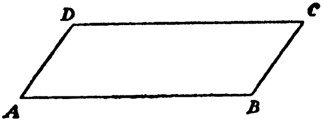


FIG. 6. A PARALLELOGRAM

EXERCISES IVH

1. What line is parallel to AB in the parallelogram above? What other lines are parallel in Fig. 6?

2. Fig. 7 shows a parallelogram with the lines forming its sides extended. Do you know, without measuring, that angle x is equal to angle y? Show that angle x equals angle z.

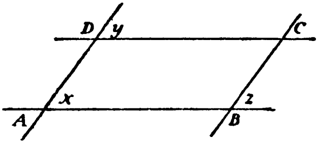


FIG. 7

3. Copy Fig. 8, and draw an angle of 50° at the point A. Call the line that you get AR. Now draw, at B, an angle of 50°. Call the line BT. Why is BT parallel to AR? Choose a point in AR; call it D. At D in AR draw an angle of 50° so that the other arm will be parallel to AB. Produce the line until it crosses BT. Call this point C. Now show that the figure ABCD is a parallelogram.

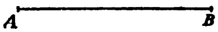


FIG. 8

4. Show at the blackboard how to draw a parallelogram. Use this method for drawing a rectangle; also for drawing a square.

5. Fig. 9 shows a parallelogram with its sides produced.

(*a*) Pick out all the angles that are equal to angle x.

(*b*) Pick out all the angles that are equal to angle y.

6. Show that $t + n = 180^\circ$. Show also that $z + u = 180^\circ$.

7. When the sum of two angles is 180° we say that they are *supplementary*, or that one is the *supplement* of the other. Find 8 pairs of angles in Fig. 9 that are supplementary.

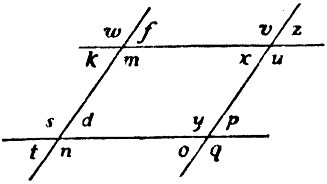


FIG. 9

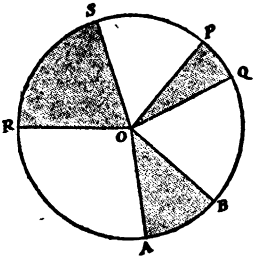
8. What is the supplement of an angle of 60° ? of 80° ? of 150° ? of 170° ? of $62\frac{1}{2}°$? of $x°$? of $y°$?

9. Two angles, a and b, are supplementary. Which of the following statements are true ? (*a*) $a+b=180$; (*b*) $180-a=b$; (*c*) $180-b=a$; (*d*) $180-a-b=0$; (*e*) $180+a=b$?

10. Angles m and n are supplementary. Write this fact as an equation—that is, in the form of the statements in question 5.

11. Draw a parallelogram ABCD in which AB is 3 in., angle B is 120°, and BC is 2 in.

46. An Important Relation which exists between the Angles at the Centre of a Circle and their Arcs.—Look at the shaded parts of this circle. Three different angles are shown at the centre of the circle: angle POQ, angle AOB, and angle ROS. Which is the smallest angle ? Which is the smallest arc, RS, PQ, or AB ? Which is the largest angle ? the largest arc ? It is clear that these angles are not equal ; neither are the arcs equal. As the angle grows larger what is true of the arc ? Let us study this more carefully.



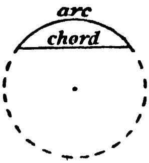
EXERCISES IVi

An experiment to compare the arcs of equal angles at the centre of a circle :

(*a*) Draw, as in the preceding figure, two equal angles at the centre of a circle. Call their arcs AB and PQ.

(*b*) Take a sheet of tracing paper and make a tracing of the arc AB. Place arc AB on arc PQ. What do you notice?

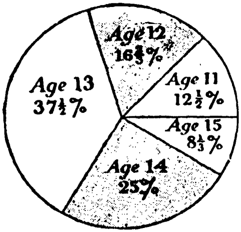
(*c*) Complete the following: If I know that two angles at the centre of a circle are equal, I know that their arcs are ——.



(*d*) Complete: If I know that two arcs in a circle are equal, then I know that their central —— are equal.

(*e*) Complete: If an angle at the centre of a circle represents one-sixth of a complete revolution, then its arc will be —— as long as the length of the circumference of the circle.

47. How to read a Circle Graph. — The headmaster of a certain school wished to know the ages of the pupils in one of the classes. A committee of the class thought that a circle graph would be a good way to report the facts to him. They sent him the following graph and table:



OUR AGES

Ages	No. of Pupils	Per Cent.
11	3	$12\frac{1}{2}$
12	4	$16\frac{2}{3}$
13	9	$37\frac{1}{2}$
14	6	25
15	2	$8\frac{1}{3}$

Exercises IVJ

1. How many pupils were in the class?

2. What fractional part of the class was 11 years old? What per cent. of the class was 11 years old?

3. What fractional part, and what per cent., of the class was 12 years old? 13 years old? 14 years old? 15 years old?

4. Why do you think they sent him the table ?

5. How many angles did they draw at the centre of the circle ? Why ? How many degrees are there in the sum of the five angles ?

6. Since the 14-year-old pupils make up one fourth of the class they will be represented by a central angle of ——.

7. (*a*) The 11-year-old group is —— of the class, and is represented by a central angle of —— of 360°, or —— degrees.

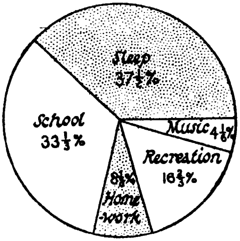
(*b*) The 12-year-old group is —— of the class, and hence it is represented by a central angle of —— of 360°, or —— degrees.

(*c*) The 13-year-old group is —— of the class, and hence it is represented by a central angle of —— of 360°, or —— degrees.

(*d*) The 15-year-old group is —— of the class, and hence it is represented by a central angle of —— of 360°, or —— degrees.

8. Use your protractor to see how accurately the committee made the graph.

9. The circle graph shown here was made by a class in a certain school. What was it made to tell ?



WHAT WE DO WITH THE 24 HOURS

(*a*) The graph shows that this class spent 37½ per cent. of the 24 hours in sleep. How many hours (on the average) did the pupils sleep ?

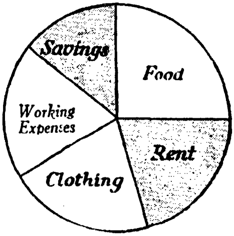
(*b*) How many hours were spent in school ? in recreation ? in homework ? in studying music ?

(*c*) Measure the angle of that part of the circle that represents the time spent in school. How large should it be ? Test each of the other angles.

(*d*) How large should the central angle that represents "recreation" be ? Test it with your protractor.

(*e*) Make a graph showing what your class does during a day (24 hours).

10. Here is the way one family spent its income. The different per cents. are omitted from the graph. See if you can find what per cent. of the income was spent for each of the items.



HOW A FAMILY SPENT ITS INCOME

11. It is thought that of incomes between £200 and £500 a year people can afford to spend 25 per cent. for food, etc., 20 per cent. for rent, 20 per cent. for clothing, 15 per cent. for working expenses, and the remainder for investments, savings, or amusements. Make a circle graph to show how a family income of £400 should be spent.

12. Large business concerns usually issue statements showing how their income is spent. A certain company engaged in transport work issued a statement which showed that out of each £100 of income £62½ went in working expenses, £4½ in interest, £5 8*s*. in taxes, £18 12*s*. in renewals, £7 in dividends on capital stock, and the remainder was deposited as a surplus fund. Represent these facts by a graph, and call it " What becomes of your £100," the title which the company used.

13. Make a circle graph showing how a family income of £150 a year is spent, if 30 per cent. is used for food, etc., 20 per cent. for rent, 15 per cent. for clothing, 10 per cent. for working expenses, and the remainder for amusements and savings.

14. Draw a circle graph representing the per cent. of the different food substances in peanuts. The approximate amounts are 7 per cent. water, 20 per cent. protein, 29 per cent. fat, 19 per cent. carbohydrates, and 25 per cent. waste.

EXERCISES IVK

Problems on Averages

1. In a class of 24 pupils 4 had 95 marks, 6 had 90 marks, 7 had 85 marks, 5 had 80 marks, and 2 had 75 marks. What was the average mark for the class? Set down your solution thus:

$$\text{Average mark} = \frac{4 \times 95 + 6 \times 90 + 7 \times 85 + 5 \times 80 + 2 \times 75}{24}$$

$$= \frac{380 + 540 + 595 + 400 + 150}{24}$$

$$= ?$$

2. On a test in mathematics the papers were marked as follows: 6 marked 90 per cent., 7 marked 85 per cent., 5 marked 80 per cent., 9 marked 75 per cent., and 3 marked 70 per cent. The average mark was —— per cent.

3. A merchant bought 42 chairs at an average price of 7*s*. 6*d*. each. He sold 6 of them for 10*s*. each, 12 of them for 12*s*. 6*d*. each, 8 of them for 8*s*. 9*d*. each, and the others for 15*s*. each. What was the average selling price?

4. The pay roll of a furniture factory shows that 16 men receive 15*s*. a day, 12 receive 12*s*. a day, 18 receive 9*s*. a day, and 8 receive 7*s*. a day. The average wage is —— shillings a day.

5. Mr Johnson sold 8 cows at £24 each, 6 at £21 each, 9 at £18 each, and 1 at £15. The average price was £——.

48. 100 per cent. Accuracy in Addition.—1. David was a very accurate reckoner. He took great pride in trying to have his reckoning 100 per cent. correct. In adding a column of numbers like the one shown here David used this method : (*a*) He added upward. Suppose the result obtained was 64. (*b*) He added downward. Suppose the result obtained was 65. The two results do not agree. (*c*) He would add upward again. Suppose the result obtained was 65. He then had agreeing answers from adding in both directions, and was reasonably certain that he was correct.

7
9
6
8
5
9
7
6
8
—

Once in a while in adding a long column David would have to repeat the addition three times to obtain two agreeing answers.

David did not consider his calculation complete until he had obtained agreeing answers from adding in both directions. Why was this a good plan ?

Use David's method to add the numbers in the column at the right. How many times did you have to add the numbers ?

7
9
5
9
8
9
6
7
8
—

2. Use the method of question 1, and find how many of these columns you can add in 4 minutes.

7	5	6	7	7	9	5	6	8	8	6	8
8	9	7	6	8	8	9	4	9	8	9	8
9	8	5	5	9	7	6	9	9	8	8	9
8	4	8	9	9	0	8	7	5	9	7	9
9	9	7	8	8	9	5	5	7	8	8	8
8	8	9	6	9	8	9	9	7	9	6	9
9	6	6	7	7	6	6	9	8	9	9	7
9	9	8	9	9	9	5	9	6	8	8	6
7	7	9	8	6	4	8	8	8	7	8	8
—	—	—	—	—	—	—	—	—	—	—	—

49. Our Number System.—(*a*) We have a *decimal* number system. The fact that we count to ten, then say eleven (ten and one), twelve (ten and two), and so on up to twenty (two tens), then twenty-one (twenty and one), and so on, makes ours a decimal system. What do we call 10 tens? 10 hundreds?

(*b*) The *base* of our number system is 10. The preceding paragraph shows how 10 is the base of our system. We believe that 10 was used as the base because of the fact that we have 10 fingers. Records show that primitive people, like children to-day, counted on their fingers.

(*c*) In our system the value of any number depends upon its *position* or *place*. In 222 the first figure represents ——; the second figure represents ——; and the right-hand one represents ——.

(*d*) Consider the number 2,347,625,834,945. This is read two billions, three hundred and forty-seven thousand six hundred and twenty-five million, eight hundred and thirty-four thousand, nine hundred and forty-five. You will notice that a billion is a million million; that is the meaning of a billion in Great Britain and in most of the northern countries of Europe, but in America and in the southern countries of Europe a billion is a thousand million. You are not likely to have to work in billions often: few people, indeed, realize how great a billion is. The following will help to make you realize what a very great number a billion is. An English mathematician, taking 4004 B.C. as the year in which Adam was created, has reckoned that if he had lived till now and had been able to count continuously at the rate of three a second, he would have counted only a little more than half of a billion.

Study carefully the table below so as to be able to read large numbers.

billions	hundred-thousand-millions	ten-thousand-millions	thousand-millions	hundred-millions	ten-millions	millions	hundred-thousands	ten-thousands	thousands	hundreds	tens	units
2	3	4	7	6	2	5	8	3	4	9	4	5

Exercises IVl

Problems with Large Numbers

1. Read the facts in the following table :

Annual Trade (Exports and Imports) of Parts of the British Empire

United Kingdom . . .	£1,350,000,000
India and Burma . . .	245,000,000
Transvaal	53,773,000
Canada	142,593,000
New Zealand	35,337,000
Jamaica	5,198,000
Western Australia . . .	15,278,000

2. How long would it take you to count enough gold sovereigns for the total of the United Kingdom's annual

trade if you counted one sovereign a second for 8 hours a day, $5\frac{1}{2}$ days a week, and 52 weeks a year ?

3. Scientists tell us that a female mosquito lays, on the average, 200 eggs, which produce 100 female mosquitoes. If in 10 days each of these has produced another 100 females, there would be 100×100, or —— females. If in 10 more days each of these has produced another 100 females, there would be $100 \times$——, or —— females. After 18 generations, or 180 days, the total number of mosquitoes hatched would be 2,000,000,000,000,000,000,000,000,000,000,000,000.

4. Have you ever heard the story of the man who made the first chequerboard ? His king was so pleased that he ordered his servants to give the man as many coins as would be obtained by allowing 1 coin for the first square, 2 coins for the second square, 4 for the third, 8 for the fourth, and so on. How many coins would be required to carry out the king's order ? (A chequerboard has 64 squares.)

REVISION EXERCISES IV

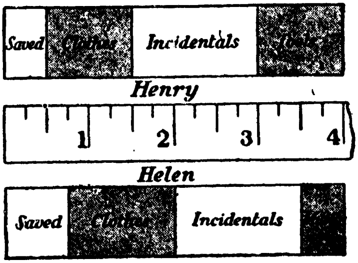
1. *A Matching Test.*—Each part of Column A belongs to some part of Column B. Match the parts in the two columns by putting the right letter in each square of Column B. For example, *a* is placed in the square in front of "A rectangle."

COLUMN A		COLUMN B
(*a*) A parallelogram with unequal sides, whose angles are right angles.	☐	Parallel lines.
(*b*) Lines that have the same direction.	☐	$\perp$.
(*c*) Lines at right angles.	☐	Supplementary angles.
(*d*) $x + y = 180°$.	☐	An angle.

Column A		Column B
(*e*) A rectangle whose sides are equal.	☐	An isosceles triangle.
(*f*) A certain amount of rotation.	*a*	A rectangle
(*g*) A triangle with two sides equal.	☐	=
(*h*) A line from the centre of a circle to the circle.	☐	Radius of a circle.
(*i*) An instrument for drawing arcs.	☐	A square.
(*j*) The longest line that can be drawn in a circle.	☐	Perpendicular lines.
(*k*) If the angles at the centre of a circle are equal.	☐	The diameter of a circle.
(*l*) The symbol for "is perpendicular to."	☐	‖.
(*m*) The symbol for " is parallel to."	☐	The compasses.
(*n*) The sign of equality.	☐	The arcs are equal.

2. The master of a trades school taught his boys to mix cement and sand, using $2\frac{1}{2}$ times as much sand as cement, or, as he said, " in the ratio of $2\frac{1}{2}$ to 1, or 5 to 2." If they use 12 bags of sand they must use —— bags of cement.

3. Some people mix sand, cement, and gravel in the proportions of 5, 3, and 2. If they use $12\frac{1}{2}$ bags of sand they will need —— bags of cement and —— bags of gravel.



4. Henry and Helen Black made this graph to tell what they did with their earnings during holidays.

(*a*) What per cent. of Henry's earnings went for savings? for clothes? for tools?

(*b*) Compare Henry's expenditures with Helen's.

(*c*) During the holidays Henry saved 20*s*. How much did he spend for each of the three items?

(*d*) Helen saved 15*s*. How many shillings did she spend on clothes? for incidentals (a general good time)?

(*e*) Why is Helen's savings bar longer than Henry's savings bar even though she saved less money?

5. During her holidays a schoolgirl earned £7 4*s*. She saved £2 5*s*., spent £1 16*s*. on clothes, £1 7*s*. on incidentals, and paid the remainder for music lessons. Make a bar graph to represent these facts.

6. All squares are ——, but all rectangles are not ——.

7. Would you use a bar graph, a line graph, or a circle graph in picturing each of the following kinds of facts?

(*a*) The weight of a baby on the first day of each month till it is 15 months old.

(*b*) The number of evening papers that a paper boy sells each day in the week.

(*c*) How Mr Stewart divided his million-pound estate among his wife, son, two daughters, and Charing Cross Hospital.

(*d*) The changes in the price of wheat during the month of August.

(*e*) A comparison of the rainfall in London, Aberdeen, Cork, and Bristol.

8. Mr Jackson's accounts showed that he had averaged £250 a year for the 21 years he had worked since leaving an elementary school. Mr Miller averaged £480 a year for the 16 years he had worked since leaving a secondary school.

(*a*) Mr Miller earned —— pounds more than Mr Jackson.

(*b*) Assuming that it was his secondary school education that enabled Mr Miller to earn more money than Mr Jackson, what financial profit did he derive from the five years he spent at a secondary school? How much was each day of secondary school worth to him? (Allow 200 days to a school year.)

9. Grace Smith was earning 9*s* a week. She took a six-months' course in a special school, after which she earned 15*s*. 6*d*. a week. Her tuition fee was £3 18*s*.; her increase in salary would pay this in —— weeks. Her salary was increased —— per cent.

10. The diameter of the earth is very nearly 7918·5 miles. In round hundreds it is —— miles; in round thousands it is —— miles. The distance round the earth is about 3$\frac{1}{7}$ times its diameter. In round numbers (thousands) the circumference is —— miles.

EXERCISES IVM

Just for Fun: Nuts to Crack

1. Here is one of the world's oldest number mysteries. The sum of the numbers in each column is 15; the sum in each row is 15; and the sum in each diagonal is 15. In the Middle Ages it was believed that this magic square would drive away disease and bring good fortune. It is still used as a charm all through the East. Can you arrange these nine numbers in a different order and still have a magic square? Or can you form a magic square that contains sixteen numbers?

4	9	2
3	5	7
8	1	6

2. Can you write one hundred, using only the nine digits and the signs of arithmetic? Here is the way Mary did it:

$$100 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 \times 9.$$

And here is the way Fred did it—but you would rather do it yourself.

3. *John.* After spending $\frac{1}{10}$ of my money I had £450 left. How much did I have at first?

Fred. You had £495 at first.

John That's just like you, Fred, to speak before you think.

4. *Grace* I doubt whether any of you can get this one: I heard of a family party at which there were

> 1 grandfather, 2 fathers, 1 grandmother, 2 mothers, 4 children, 3 grandchildren, 1 brother, 2 sisters, 2 sons, 2 daughters, 2 married men, 2 married women, 1 father-in-law, 1 mother-in-law, and 1 daughter-in-law.

Marie. There were 27 persons present.

Grace. Oh, no! You are nowhere near it.

5. A man had five pieces of chain, each having 3 links. "How much would you charge me to make these five pieces of chain into one piece?" he asked the blacksmith.

"That's not expensive. I charge twopence to cut a link and twopence to weld a link." "I see; that's 20 pence." "Oh, no," said the blacksmith.

6. Mary was carrying a basket of eggs to market. A farmer driving an unruly mule struck the basket and broke all the eggs.

"Never mind the loss," he said, "I'll pay for the eggs. How many did you have?" "I'm not sure," she said, "but when I counted them by twos there was one egg left over; when counted by threes there was one left over; when counted by fours there was one left over; but when counted by fives there were no eggs left over." The farmer was a bit perplexed at this, but soon marshalled his wits and said to Mary: "You had —— eggs."

7. Professor Rackbrane used to give this little poser to his classes: A man sold his farm for £4000, which is what he paid for it Then he bought it back for £3500, and later sold it for £4500. How much did he gain?

8. Mr Cute had £2 and wanted £2 10*s*. He pawned the £2 for £1 10*s*. and then sold the pawn-ticket for £1; then he had his £2 10*s* Did anyone lose on the transaction?

RUNNING THE SUBTRACTION 'HURDLE' TEST

You should practise this test until you can 'take' these hurdles in 10 minutes. You should practise once a week. More than one error on each hurdle knocks it down. You must finish without knocking a single hurdle.

Hurdle 1

(*a*)	(*b*)	(*c*)	(*d*)
40·00	80·00	100·00	61·5
17·94	0·64	72·05	17·68

Hurdle 2

(*a*)	(*b*)	(*c*)	(*d*)
$7\frac{3}{4}$	$8\frac{1}{2}$	$2\frac{1}{2}$	16
$2\frac{1}{4}$	$5\frac{1}{2}$	$1\frac{1}{2}$	$9\frac{1}{4}$

Hurdle 3

(*a*)	(*b*)	(*c*)	(*d*)
7	8	4 ft. − 6 in.	3 hr. − 20 min.
$2\frac{2}{3}$	$2\frac{4}{5}$		

Hurdle 4

(*a*)	(*b*)	(*c*)	(*d*)
200 − 0·42	$6\frac{1}{2}$	$8\frac{1}{4}$	$10\frac{1}{3}$
	$3\frac{3}{4}$	$5\frac{3}{4}$	$6\frac{2}{3}$

Hurdle 5

(*a*)	(*b*)	(*c*)	(*d*)
92,304	70,004	80,808	46·00
78,998	39,099	60,909	12·45

PRACTICE FOR ACCURACY AND SPEED

Test 7. Short Division

3)174	9)846	4)192	8)760	8)688
4)296	7)693	6)570	6)246	7)511
5)285	6)504	5)490	4)304	9)630
9)855	7)252	6)534	3)609	5)510

Test 8. Multiplication with Two-digit Numbers

2468	5973	7395	26
92	37	85	268
6482	5379	2684	85
64	29	73	307

Test 9. Adding Single Columns

You should check your work by adding each column a second time, or a third time if necessary. Repeat your work until you obtain two agreeing results for each example.

2	1	7	2	3	8	7	4
9	8	8	7	3	8	1	3
4	9	2	9	8	1	4	2
1	5	1	6	3	9	4	1
9	6	3	5	1	8	4	3
2	9	6	2	5	2	1	5
7	0	8	6	7	5	3	7

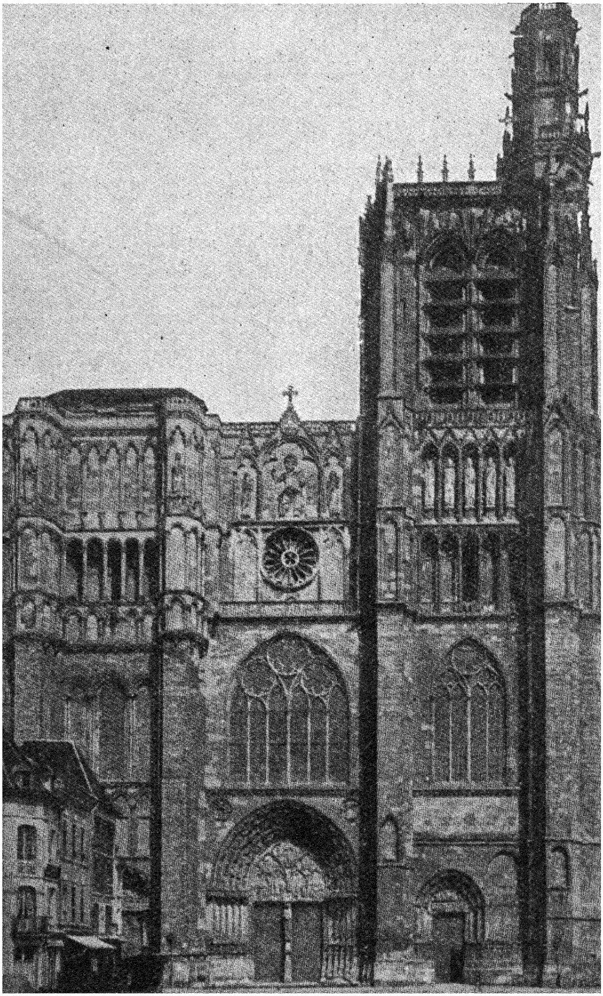
9	8	4	2	9	1	9	3
5	2	4	5	7	5	3	9
3	4	4	3	9	7	1	5
3	8	9	9	0	5	5	7
6	5	8	8	5	5	3	3
8	4	6	9	4	4	6	2
7	8	4	4	8	4	2	6
—	—	—	—	—	—	—	—

9	9	9	7	5	5	5	9
7	6	3	7	1	7	5	5
3	3	7	6	7	4	9	2
7	4	5	6	6	9	5	4
5	3	3	3	8	8	4	5
6	8	7	7	6	3	7	1
3	6	6	6	9	3	8	6
—	—	—	—	—	—	—	—

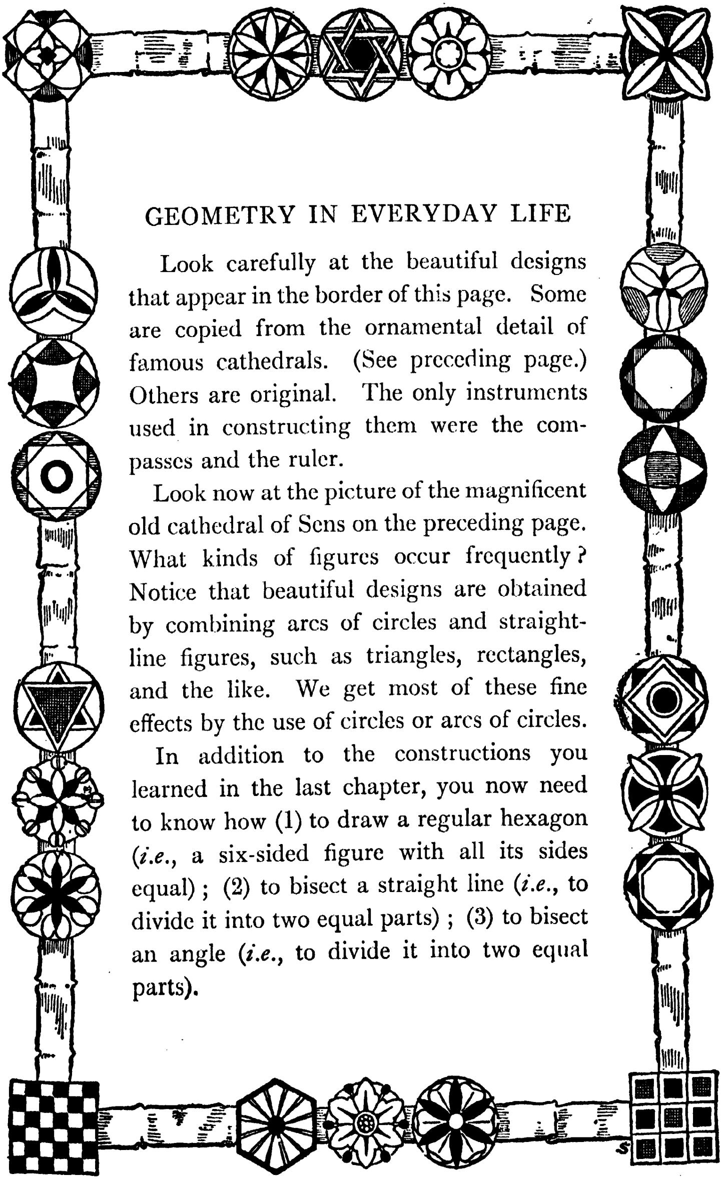
When you pass Test 9 turn to Test 10 on p. 137. The next test is for practice at home.

CHAPTER V

GEOMETRY IN ARCHITECTURE—INTEREST AND OTHER PROBLEMS



Note the many geometric figures in this famous old cathedral at Sens.



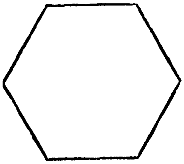
GEOMETRY IN EVERYDAY LIFE

Look carefully at the beautiful designs that appear in the border of this page. Some are copied from the ornamental detail of famous cathedrals. (See preceding page.) Others are original. The only instruments used in constructing them were the compasses and the ruler.

Look now at the picture of the magnificent old cathedral of Sens on the preceding page. What kinds of figures occur frequently? Notice that beautiful designs are obtained by combining arcs of circles and straight-line figures, such as triangles, rectangles, and the like. We get most of these fine effects by the use of circles or arcs of circles.

In addition to the constructions you learned in the last chapter, you now need to know how (1) to draw a regular hexagon (*i.e.*, a six-sided figure with all its sides equal); (2) to bisect a straight line (*i.e.*, to divide it into two equal parts); (3) to bisect an angle (*i.e.*, to divide it into two equal parts).

50. How to draw a Hexagon by using a Circle.—The figure at the left is a *regular hexagon*. Its sides are all equal, and its angles are all equal. Since the regular hexagon is used so often in designs we shall learn how to draw one by making use of a circle.



A REGULAR HEXAGON

(*a*) Fig. 1 shows a circle with radius OA. Make an enlarged copy of Fig. 1. Mark a point A on the circumference. With the same radius begin at A and mark off a series of small arcs round the circle, each one cutting the circle. If you are very careful you will find that the sixth arc intersects the circle at A. Your work will look like Fig. 2.

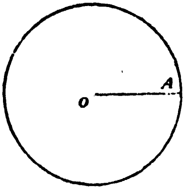


Fig. 1

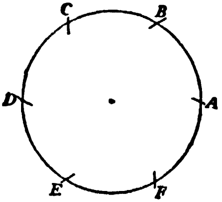


Fig. 2

(*b*) Now join the successive points, A to B, B to C, and so on. Your work should look like Fig. 3.

(*c*) Look at Fig. 4. Remember that AB is equal to the length of the radius. Show that in triangle OAB all three sides are equal.

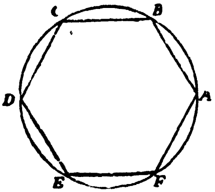


Fig. 3

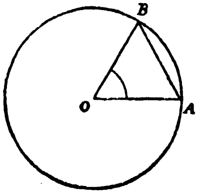


Fig. 4

(*d*) Show that the angle at O is 60°.

(*e*) How many angles can you draw at the centre of the circle each the same size as angle AOB ?

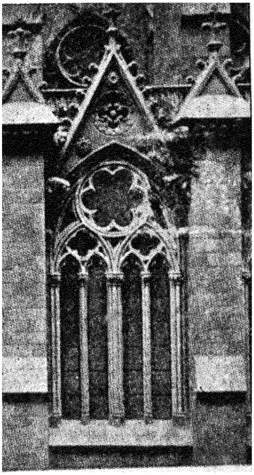
(*f*) How many triangles like the triangle AOB could you draw in the circle shown in Fig. 4 ?

(*g*) When you 'step round a circle' with the compasses opened the same distance as the length of the radius, why should the sixth arc cut the circle at the point at which you started ?

Exercises Va

1. Say how to construct a regular hexagon.

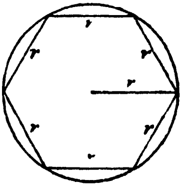
2. Show where the hexagon is used in the cathedral window shown here.



3. Show that each side of the hexagon may be represented by r. (r = the number of units in the radius of a circle.)

4. Miss Smith asked her class to tell her the length of the perimeter of (distance round) a regular hexagon. Some of the pupils said it was "$r+r+r+r+r+r$"; others said it was "six times the radius"; still others said it was "$6r$"; and one girl said it was "six times the diameter." Which answers were correct ?

5. If p represents the number of units in the perimeter, and r the number of units in one side of a hexagon, what does $p=6r$ mean ? Give p when r is 2·5.

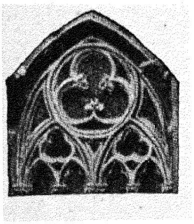


6. Draw a regular hexagon, making each side 2 in

7. Can you draw a regular hexagon without using the compasses ? It is possible to draw hexagons by using a ruler and a protractor.

Can you ? If you can, construct a regular hexagon, making each side 2 in.

51. How to draw an Equal-sided Triangle by Means of a Circle.—We frequently use equal-sided triangles in making designs. Look at the picture of the cathedral window shown here. Find the design at the apex of the window. What kind of figure would be formed by joining the centres of these circles ?



A triangle with all sides equal is an *equilateral* triangle.

Exercises Vb

1. Make an enlarged copy of Fig. 1. Instead of joining points A, B, C, etc., in turn, as you did in drawing a hexagon, join every other point, as in Fig. 2. What kind of figure do you get ?

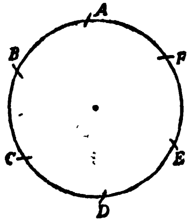


Fig. 1

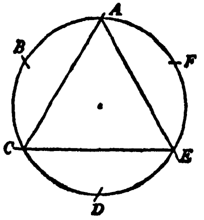


Fig. 2

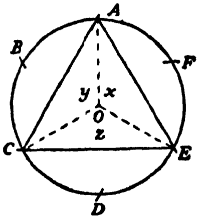


Fig. 3

2. Fig. 3 is the same as Fig. 2, except that points A, C, and E are joined to the centre of the circle, O. How many degrees are there in the angle AOE ? In angle AOC ? In angle COE ? What is the sum of these angles ?

3. Mary said she could construct an equilateral triangle as follows: (*a*) construct a circle; (*b*) 'step off' arcs

with the compasses set equal to the diameter of the circle. Show whether Mary was or was not correct.

4. Can you discover a quicker method of drawing an equilateral triangle ?

52. We need to know how to bisect a Straight Line.—In drawing designs you often need to use a line half as long as some other line. Up to this time you have probably found half of a straight line by estimating with the compasses, or by measuring with the ruler. There is a very accurate way of dividing a straight line into two equal parts (*bisecting* a straight line) with the compasses and ruler. It is used again and again in practical drawing.

EXERCISES VC

1. We wish to bisect the line AB in Fig. 1.

(*a*) With A as a centre draw an arc, such as CD.

(*b*) With B as a centre, and with the same radius as before, draw an arc, such as EF.

(*c*) Notice that these arcs intersect at two points, X and Y. See Fig. 1.

(*d*) Join X to Y. See Fig. 2. XY intersects AB at the point M. M is the middle point of AB ; that is, XY bisects AB.

(*e*) Is AB in Fig. 2 accurately bisected ? Check with your compasses ; with your ruler.

(*f*) Show by using your protractor that XY in Fig. 2 is a *perpendicular bisector* of AB.

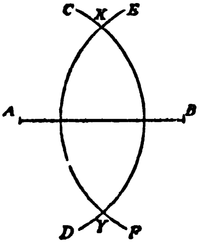


FIG. 1

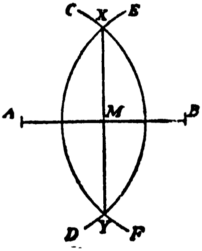
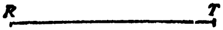


FIG. 2



2. Make an enlarged copy of the line RT and bisect it. Could the radius of the arcs be any length?

3. Show how to divide RT into four equal parts without actually measuring it.

4. Can this method be used to divide a line into six equal parts? eight equal parts? ten equal parts? sixteen equal parts?

53. How to draw a Square by Means of a Circle. We are learning how to draw each of the common, regular, straight-line figures. We have learned how to draw the regular six-sided figure (hexagon) and the three-sided figure (equilateral triangle). Next we shall learn how to draw a square by means of a circle.

(*a*) Fig. 1 shows a circle with a diameter drawn. Make an enlarged copy of this.

(*b*) In the circle you drew for (*a*) draw a diameter perpendicular to the diameter AC. See Fig. 2. (Use only compasses and ruler.) Join the four points A, B, C, and D. Your work should look like Fig. 3. What kind of figure is ABCD? Why?

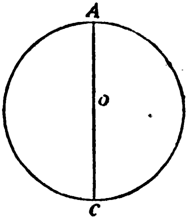


Fig. 1

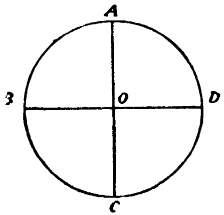


Fig. 2

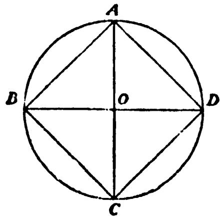


Fig. 3

Exercises Vd

1. Draw a square in a circle which has a 2-in. radius.
2. Draw a square in a circle which has a 3-in. diameter.

3. Is a square a *regular* straight-line figure? Can a straight-line figure have equal sides and unequal angles? Show that a triangle may or may not be a *regular* figure.

4. Copy Fig. 4. What radius was used to draw the arcs?

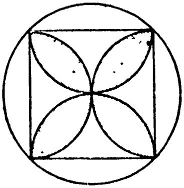
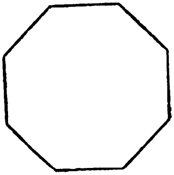
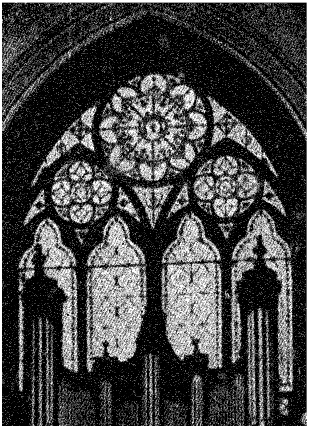


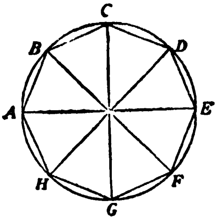
FIG. 4

54. How to construct a Regular Octagon (Eight-sided Figure).—Many geometric figures are based upon regular octagons—that is, they make use of figures having eight equal sides and eight equal angles. Look at this picture of a cathedral window. Do you see a figure in it that is based upon an octagon? Which one? Now look at the figure beside the picture. Why is the figure inside the circle called an octagon?



AN OCTAGON





To draw a regular octagon we have to know how to find the eight points shown on the circle. We can do this by constructing eight equal angles at the centre of the circle. We know how to construct four right angles at the centre of a circle. Each of these right angles must be divided into two equal angles. In other words, we must *bisect* each of the right angles. The next section shows how.

55. How to bisect Angles.—James wished to bisect the angle shown in Fig. 1. He learned to do it as follows:

(*a*) He set one point of the compasses at O, and marked off an arc cutting the sides of the angle, OA and OB, at R and T. His work then looked like Fig. 2.

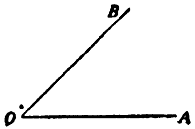


FIG. 1

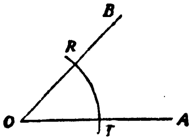


FIG. 2

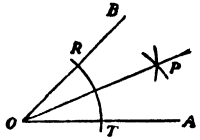


FIG. 3

(*b*) With R and T as centre, and with a radius of convenient length, he drew arcs cutting each other at P, as in Fig. 3.

(*c*) He drew a straight line from O through P. This line PO bisects (divides into two equal parts) the angle AOB. Hence, it is called the *bisector* of angle AOB.

Exercises Ve

1. Make an enlarged copy of Fig. 3. With a sharp knife cut out the figure. Fold it over on OP so as to show that the angle has been bisected.

2. Draw an angle and bisect it. Check the construction with the protractor.

3. Show how to construct an angle of 45° with the compasses. Check your construction with a protractor.

4. Show (*a*) that the diagonals of a square are perpen-

dicular to each other, and (*b*) that they bisect the angles of the square.

5. Jane said that she could construct an angle of 60° without using the protractor. Do you see how she could do it ?

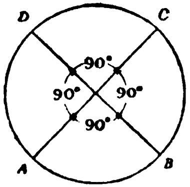


FIG. 4

6. Joseph drew a parallelogram, ABCD, and a diagonal AC. He concluded that AC bisected angle A and angle C. Was his conclusion correct ? Illustrate.

7. Fig. 4 shows how you begin to construct a square. What would you still have to do if you wanted to draw a square ?

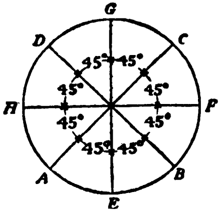


FIG. 5

8. Fig. 5 shows each of the angles in Fig. 4 bisected. How many angles are there now ? How many points are marked on the circle ?

9. Make an enlarged copy of Fig. 5, and draw lines AE, EB, etc. The resulting figure is a regular octagon.

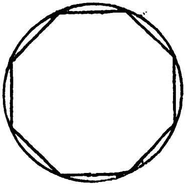


FIG. 6

10. Show how Fig. 6 is obtained from Fig. 5. What parts of Fig. 5 did you have to erase ?

11. Draw a regular octagon in a circle whose radius is 5 cm.

12. The two tests to tell whether an 8-sided figure is a *regular* octagon are :

(*a*) The sides must all be equal.
(*b*) The angles must all be equal.

Apply these tests to your drawing for question 11.

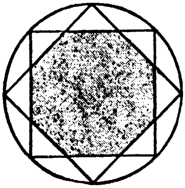


FIG. 7

13. Make a larger copy of Fig. 7.

If it is necessary to draw lines which do not appear here, draw them lightly in pencil, and later erase them.

14. What is the meaning of the statement, "A gun barrel is octagonal" ?

56. Paying for the Use of Money: Interest.—Fred Armstrong needed £25 to finish paying his way through the Secondary School to which he had been sent some years before. His uncle lent him the money on the understanding that Fred was to pay £1 for the use of the money for 1 year. The £1 was *interest.*

EXERCISES VF

1. Fred paid £1 interest for the use of £25 for 1 year. The interest was —— per cent.

2. If Fred's uncle received £1 for the use of his £25 for 1 year his rate of interest was —— per cent. What would the rate of interest have been if he had received £2 for the use of the money for 1 year ? If he had received 15*s.* ?

3. If Fred had kept the money for 2 years the interest would have been —— pounds ; if he had kept it only 6 months the interest would have been —— shillings.

4. What would you pay for the use of £100 for 1 year at 6 per cent. ? At 5 per cent. ? At 4½ per cent. ? At 3 per cent. ?

5. If you lent £50 for 1 year at 5 per cent. how much would you expect to receive as interest ?

6. Complete : At 6 per cent. the interest on £50 for 1 year is ——; the interest for 6 months is ——; the interest for 1 month is ——.

7. Complete : One year's interest

(*a*) On £500 at 6 per cent. is £——
(*b*) On £800 at 5 per cent. is £——.
(*c*) On £650 at 6 per cent. is £——.

(*d*) On £760 at 4 per cent. is £——.
(*e*) On £1250 at 5 per cent. is £——.
(*f*) On £3200 at 6 per cent. is £——.
(*g*) On £875 at 4 per cent. is £——.
(*h*) On £125 at 5 per cent. is £——.

8. Show that the interest on £1500 for 2 years at 6 per cent. is £0·06 × 1500 × 2.

9. Find the interest on £645 for 1 year at $4\frac{1}{2}$ per cent.

10. What is the interest on £645 for 2 years at $5\frac{1}{2}$ per cent.?

11. Find the interest on £875 for 2 years at $5\frac{1}{2}$ per cent.

12. John Smith borrowed £450 from Jack Jones at $6\frac{1}{2}$ per cent. How much did he owe Mr Jones at the end of 1 year?

13. Mr Cox borrowed £5000 from the bank for 3 months at 6 per cent. How much interest did he have to pay?

14. What is the interest on £750 for 1 year at 5 per cent.? For 2 years? For 6 months?

15. Show that the interest on any sum of money for 1 year at 6 per cent. is 0·06 times that sum.

16. Examine the following statement to see if it is true: The interest on any sum of money at 5 per cent. for 4 years is 0·05 times that sum times 4.

17. The sum of money borrowed is called the *principal*. Write a rule for finding the interest if you know the principal, the rate, and the time.

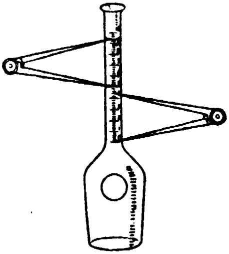
18. Show that the following is a helpful way to remember how to find interest:

$$\textit{interest} = \frac{\textit{principal} \times \textit{rate} \times \textit{time in years}}{100}.$$

19. Find the interest when the principal is £540, the rate is 6 per cent., and the time is 1 year.

20. Find the interest when the principal is £65, the rate is 5 per cent., and the time is 4 months.

57. Testing Milk.—A test for the amount of butter fat in milk may be made in bottles similar to the one shown here. The cream collects in the neck of the bottle and can be measured on the scale with a pair of dividers. The picture shows a test yielding 4 per cent. butter fat.



EXERCISES VG

1. A farmer took a can of milk weighing 280 lb. to the creamery. The test for this milk showed that it contained 3·6 per cent. of butter fat. How much did he receive for this butter fat at 1*s*. 2*d*. per pound ?

2. Mild Rose, a Jersey cow, yielded 16,482 pounds of milk in one year. From this amount of milk 992 pounds of butter fat were obtained. What per cent. of the milk was butter fat ?

3. Molly Moo, a Holstein cow, yielded an average of 14,240 pounds of milk per year for four years. If her milk when tested yielded 3·6 per cent. butter fat, how many pounds of butter fat did this cow produce in the four years ? How much was this butter fat worth at 1*s*. 2*d*. per pound ?

4. Tom tested five cows for butter fat, as follows :

Cow	Pounds of Milk per Day	Per Cent. of Butter Fat
Spotty	32	3·8
Star	30	3·6
Rose	26	4·1
Fence-jumper . . .	36	4·0
Maud	27	3·2

Tom's father wanted to sell two cows. On the basis of this test which cows should be sold ?

58. Reckoning the Selling Prices of Goods.—Mr Tomkins runs a fancy bazaar. He engaged a girl assistant, Lily, to help him fix the prices for the different articles. He taught her to calculate the selling price of the various articles as follows :

(*a*) Find 40 per cent. of the cost of each article, and
(*b*) Add that to the cost.

Exercises VH

1. Use the method given above to fix the selling price of :

		Cost Price
(*a*)	Fountain pens	2*s*. 1*d*.
(*b*)	School bags	3*s*. 4*d*.
(*c*)	Toy wagons	1*s*. 8*d*.
(*d*)	Pencil-sharpeners	2½*d*.
(*e*)	Large dolls	4*s*. 2*d*.
(*f*)	Paint-boxes	2*s*. 6*d*.
(*g*)	Rubber balls	5*d*.
(*h*)	Doll's tea-sets	2*s*. 11*d*.

2. During one week the sales in the fancy bazaar amounted to £73. Lily had found that the net profit averaged 12½ per cent. of the selling price. What was the net profit for the week ?

3. For a Christmas sale Mr Tomkins bought £115 worth of goods, which Lily marked to sell at 40 per cent. above cost. How much did she expect to receive for them ? What was the net profit if all the goods were sold ?

Exercises VI

Problems about Health, etc.

1. What is the average length of human life ? A certain European city has kept vital statistics since about the year

1500. An examination of these statistics reveals the following facts:

In the 1500's the average length of life was 21 years.
In the 1600's the average length of life was 26 years.
In the 1700's the average length of life was 34 years.
In the 1800's the average length of life was 40 years.

The average length of life in the 1800's was an increase of —— per cent. over the average length in the 1500's.

2. In a certain school 55 children were examined by the school doctor. His report showed the following:

Defects	Number of Children	Per Cent. of Total Number
Eyes	5	?
Hearing	2	?
Breathing	44	?
Teeth	50	?
Other defects . . .	5	?

What per cent. of the children had each defect?

3. Is smoking harmful? A committee was appointed to find out whether smokers were as successful in being chosen for football teams as non-smokers. The results of the investigation are given here:

Institution		A	B	C	D	E	F
Number competing for places	Smokers . .	11	10	28	28	10	6
	Non-smokers .	19	25	17	15	15	26
Number successful	Smokers . .	2	4	7	11	7	0
	Non-smokers .	11	17	14	10	12	15

Find what per cent. of the smokers were chosen to play in the teams. What per cent. of the non-smokers were selected?

4. In 1922 in Great Britain there were 2765 fatal accidents on the roads and streets. These were accounted for as follows :

Motor omnibuses	193
Tramcars	111
Other mechanically propelled vehicles	2026
Horse-drawn vehicles	265
Cycles	170
Total	2765

How many people on an average were killed by road traffic every day ? What per cent. of the fatal accidents were due to motor omnibuses ? To tramcars ? To horse-drawn vehicles ? To cycles ?

5. It is estimated that there are 1,500,000,000 people in the world. 32,250,000 people die each year. What per cent. of the world's population dies each year ? How many die each day ? each hour ? each minute ?

6. A record was kept of the reasons given for taking boys away from the secondary schools in a certain area before they reached the age of 16. The figures when expressed as percentages were as follows :

Ill health	1·5	per cent.
Had to go to work (difficulties at home)	39·5	,,
Parents disappointed at boy's progress	20	,,
Inability to pass any examination	12	,,
Parents leaving the district	3	,,
Disliked school	2	,,
Other reasons	15·5	,,
Reason unknown	6·5	,,

What were the chief reasons ? In a school of 650 boys 25 boys left in one year before they reached the age of 16. How many of these left because their parents were disappointed at their progress ? because of their inability to pass any examination? because they had to go to work?

Make a bar graph or a circle graph to illustrate the facts given in the list above

EXERCISES VK

Problems about the Farm

1. What is the difference in the earning value of two hens, one laying 290 eggs a year and the other 80 eggs a year, if the average price of eggs is 2*s*. 6*d*. per dozen?

2. It is estimated that hens average 90 eggs a year each, but by careful selection an average of 150 eggs a year can be obtained. What would this difference amount to for a year on a chicken farm with 200 hens when eggs are 2*s*. 3*d*. a dozen?

3. If a cow averages daily 2 gallons of milk which is 4·1 per cent. butter fat, how much butter fat does she produce per year?

4. For £35 a farmer can buy a cow that gives 2 gallons of milk a day, or for £50 a Jersey cow that gives 3 gallons of milk a day. If the first cow's milk is 2·2 per cent. butter fat and the Jersey's 4·2 per cent., and the butter fat averages 2*s*. 3*d*. a pound the year round, which cow will prove to be the better investment by the end of the first year? (Take 1 pint as equivalent to $1\frac{1}{4}$ lb.)

5. Mr Wilkins, a farmer, can increase the weight of each ear of corn 1·5 ounces by proper selection of seed. What will the increase amount to on a 10-acre field averaging 6400 stalks per acre, 1 ear for every stalk, when corn is selling for x shillings per bushel? (1 bushel of ear corn weighs 70 lb.)

6. The table below gives the average price per quarter of British corn from 1914 to 1922. Draw a suitable graph to illustrate the fluctuations in price.

Year	1914	1915	1916	1917	1918	1919	1920	1921	1922
Price	34*s*. 11*d*.	52*s*. 10*d*.	58*s*. 5*d*.	75*s*. 9*d*.	71*s*. 11*d*.	72*s*. 11*d*.	80*s*. 10*d*.	71*s*. 6*d*.	47*s*. 10*d*.

7. A farmer sold a team of horses for £98, but did not receive his pay for them until 1 year 10 months after the sale. He had at the same time a cash offer of £90 for them. Did he gain or lose by the sale, and how much, money being worth 6½ per cent.?

Exercises VI

Buying at a Discount

1. The Lost and Found Department at the Oldtown Grammar School had a sale of second-hand books. This advertisement was posted on the school notice-board:

A Big Saving on Slightly Used Books
See Price-list Below

Kind of Book	Cost when New	Present Condition	Discount Allowed	Sale Price
1 Geography	2*s*. 6*d*.	Good	20 %	
3 Spelling books	4*d*.	Poor	50 %	
1 History	3*s*. 4*d*.	Fair	45 %	
3 Algebra	2*s*. 8*d*.	Poor	75 %	
5 Latin grammar	3*s*. 0*d*.	Fair	25 %	
1 French prose	2*s*. 11*d*.	Badly worn	80 %	

Fill in the sale price of each of the books. The cost when new is the price per single copy.

2. The fairy cycle which Mary wanted was marked £5 2*s*. 6*d*. before Christmas. A few days after Christmas Mary saw the fairy cycle in the shop window, and on it a ticket which said "20 per cent. discount." What was the price of the fairy cycle then?

3. The *net* price is the price after the discount or

deduction has been made. What was the net price of Mary's fairy cycle ?

4. Find the discount and the net price of each of the following articles :

(*a*) A bicycle marked £9 less 15 per cent.
(*b*) A fretsaw outfit marked 17*s*. 6*d*. less 20 per cent.
(*c*) A toy engine marked 11*s*. 8*d*. less 35 per cent.
(*d*) A doll marked 16*s*. 8*d*. less 12 per cent.

5. What is the net cost of 60 copies of a mathematics book sold at a discount of $16\frac{2}{3}$ per cent. off the marked price, 3*s*. 6*d*. ?

EXERCISES VM

Problems for Practice[1]

I. *Multiplication :*

1. 305×407
2. 250×607
3. 330×400
4. 708×897
5. 789×968
6. 520×850
7. 480×250
8. 908×97
9. $4\cdot02 \times 80\cdot9$
10. $47\cdot5 \times 30$
11. $8\cdot40 \times 57\cdot9$
12. $7\cdot08 \times 9\cdot69$
13. $8\cdot04 \times 2\cdot04$
14. $50 \times 0\cdot05$
15. $\cdot709 \times 1000$
16. $0\cdot08 \times 80$
17. $12\frac{1}{4} \times 17\frac{1}{2}$
18. $156\frac{1}{2} \times 3\frac{1}{5}$
19. $12\cdot6 \times 8\frac{1}{3}$
20. $25\cdot4 \times 17\frac{3}{4}$
21. $156\frac{1}{4} \times 8\frac{1}{3}$
22. $16\frac{1}{2} \times 16\frac{1}{2}$
23. $7\cdot2 \times 7\cdot02$
24. $3\frac{2}{3} \times 7\cdot5$

II. *Division :*

1. $14{,}160 \div 72$
2. $31{,}095 \div 79$
3. $45{,}760 \div 92$
4. $66{,}666 \div 98$
5. $138{,}500 \div 189$
6. $151{,}080 \div 261$
7. $201{,}695 \div 298$
8. $378{,}005 \div 421$
9. $253\cdot125 \div 45\cdot2$
10. $3237\cdot3 \div 4\cdot95$
11. $2409 \div 275$
12. $47\cdot8923 \div 6\cdot07$
13. $1\cdot75088 \div 2\cdot48$
14. $17{,}812\cdot4 \div 3\cdot08$
15. $2\cdot90304 \div 5\cdot04$
16. $17{,}280 \div 0\cdot12$
17. $3 \div 1\frac{2}{3}$
18. $150 \div 12\frac{1}{2}$
19. $142\frac{6}{7} \div 28\frac{4}{7}$
20. $3\frac{1}{3} \div 66\frac{2}{3}$
21. $62\frac{1}{2} \div 37\frac{1}{2}$
22. $1\frac{2}{5} \div 1\cdot4$
23. $4 \div 0\cdot04$
24. $0\cdot04 \div 4$

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[1] TO THE TEACHER. The exercises on this page are not a part of the series of *timed* practice exercises on which the pupil keeps a record of individual improvement. They are supplementary practice materials for general class use.

III. *Mixed operations :*

1. $\frac{1}{3} + \frac{7}{10} + \frac{5}{12}$
2. $\frac{7}{10} + \frac{8}{15} + \frac{1}{3}$
3. $\frac{4}{5} - \frac{3}{10} + \frac{1}{2}$
4. $\frac{4}{5} \times 25 \times \frac{1}{2}$
5. $12\frac{1}{2} \times 3\frac{1}{5}$
6. $6\frac{3}{4} \div \frac{7}{12}$
7. $500 \div 33\frac{1}{3}$
8. $\frac{3}{4} + \frac{1}{6} + \frac{5}{8}$
9. $0{\cdot}02 + \frac{1}{5}$
10. $2{\cdot}008 \div 0{\cdot}02$
11. $66\frac{2}{3} \times \frac{3}{2}$
12. $24\frac{1}{2} \div 6{\cdot}25$

EXERCISES VN

Little Examinations

1. *Correct Copying.*—How many of these numbers can you copy correctly in 2 minutes ? Look carefully at each number so that you can keep it in mind while you write it.

916	384	4258	7684	76,846
828	769	9784	9867	90,209
758	810	8305	5968	82,907
592	160	9008	6090	90,008
501	796	8295	1957	65,565

2. *Changing Common Fractions to Decimals.*—Express each of these fractions in the form of a decimal, correct to hundredths :

(*a*) $\frac{2}{3}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{10}$ $\frac{3}{4}$ $\frac{3}{5}$ $\frac{3}{7}$ $\frac{3}{8}$

(*b*) $\frac{4}{5}$ $\frac{5}{8}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{2}{7}$ $\frac{2}{9}$ $\frac{5}{12}$ $\frac{7}{12}$ $\frac{3}{10}$ $\frac{5}{9}$ $\frac{1}{6}$ $\frac{3}{16}$

3. State what change (coins and notes) you would be likely to get for each of the following :

(*a*) A 7*s*. 3*d*. purchase paid for out of a pound note.
(*b*) A £3 4*s*. 7*d*. purchase paid for out of a five-pound note.
(*c*) A £1 14*s*. purchase paid for out of 2 pound notes.
(*d*) A 17*s*. 1½*d*. purchase paid for out of a pound note.
(*e*) A £7 11*s*. 6*d*. purchase paid for out of a ten-pound note.
(*f*) Purchases of £1 7*s*. 3*d*., 17*s*. 6*d*., 4*s*. 3*d*. paid for out of 3 pound notes.
(*g*) Purchases of £2 11*s*. 6*d*., 19*s*. 4*d*., 7*s*. 11*d*., £5 11*s*. 7*d*., paid for out of a ten-pound note.

REVISION EXERCISES V

1. A regular hexagon is a —— -sided figure with equal —— and equal ——.

2. If a regular hexagon is drawn in a circle whose radius is r units, we know that the length of each side of the hexagon is —— units.

3. To bisect a straight line means to find —— of it, or to —— it into —— equal parts.

4. Show on the blackboard how to bisect a straight line.

5. If s represents the length of each side of a regular figure, and p represents its perimeter, tell what is meant by the equations $p = 3\ s$, $p = 4\ s$, $p = 6\ s$, and $p = 8\ s$.

6. Each angle at the centre of a regular hexagon contains —— degrees; each angle at the centre of a regular octagon contains —— degrees.

7. Chords are measured in —— units, while angles are measured in —— units.

8. Make a drawing in which a line CB is a bisector of an angle DCE.

9. Name five kinds of geometric figures found in architecture.

10. How could you draw a regular sixteen-sided figure? Thirty-two sided figure? Twelve-sided figure?

11. Fig. 1 is a design for oilcloth. What kind of figures does it contain?

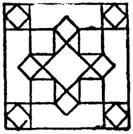


FIG. 1

12. Make a drawing in which a line XY is a perpendicular bisector of another line MN.

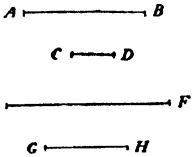
13. Two parallel lines, MN and GH, are cut by a line RS, forming the corresponding angles, x and y. Illustrate by a free-hand drawing.

14. In making a bar graph John used a bar $1\frac{7}{8}$ inches long to represent £450. How long should he make a bar to represent £510?

15. Joyce measured some cloth for schoolroom curtains with a tape-measure which she thought was 1 yard long. She measured off 8½ yards. Later she found that the tape was only 2 feet 10 inches long. What was the true length of the piece she had measured off ?

16. At A on the line AB below draw AC, making angle BAC equal to 60° ; then draw at B a line parallel to AC. Use the protractor.

17. Can you find two straight lines in this figure whose lengths are in the ratio of 3 to 2 ? 1 to 2 ? 3 to 4 ? 1 to 3 ? Use your ruler.



18. In this figure CD is 50 per cent. of —— ; AB is 75 per cent. of —— ; GH is 66⅔ per cent. of —— ; EF is 33⅓ per cent. of —— ; and AB is 50 per cent. of ——.

19. Change 2 in. to centimetres, 5 cm. to inches, 1 m. to centimetres, 1 m. to inches, and 1 ft. to centimetres

20. What can I find if I know (*a*) the side of a square ? (*b*) the perimeter of a square ? (*c*) the cost and selling price of an article ? (*d*) what a boy earns and what he spends ? (*e*) the number of games Aston Villa have played and the number they have won ?

Practice for Accuracy and Speed

Test 10. Addition

5036	2096	8847	9896	6394	3739
9473	7943	6094	5947	8835	3496
2760	8592	6920	6244	9773	1046
9605	3056	5409	5338	7607	1227
3508	7709	6205	9606	3269	2309
7934	9095	2684	6032	4209	6794
6327	7489	5292	7468	7096	3283

Test 11. Practice in Division

58)2726 34)1666 46)2714

86)5848 56)4368 63)5544

Test 12. Addition

·56	968·09	54·58	48·49	·48
1·1	5·83	9·87	7·45	8·59
·75	·58	4·63	604·09	669·97
1·01	6·25	9627·25	9·75	2·98
·33	97·64	·92	·53	·65
1·69	3·47	8·57	9366·96	8477
·63	·49	2·46	5·08	9·55
·07	5·6	297·94	·63	8·06
·49	986·87	·67	6·24	346·45

Test 13. Division

47)27,589 96)56,064 48)32,304

27)2403 39)3783 78)7566

Test 14. Fractional and Percentage Equivalents

See if you can do the ten examples, with not more than one error, in four minutes.

In each of the following exercises select the correct parts :

1. 60 per cent. equals ($\frac{60}{100}$, 0·06, $\frac{6}{10}$, $\frac{3}{4}$, $\frac{3}{5}$).
2. 40 per cent. equals ($\frac{4}{100}$, 0·04, $\frac{40}{100}$, ·040, $\frac{2}{5}$).

3. 75 per cent. equals ($\frac{75}{100}$, 0·75, $\frac{750}{1000}$).
4. $12\frac{1}{2}$ per cent. equals (1·25, 0·125, $1\frac{1}{8}$, $\frac{1}{8}$).
5. $37\frac{1}{2}$ per cent. equals ($\frac{3}{8}$, 0·375, $\frac{375}{1000}$).
6. $\frac{5}{8}$ equals (0·62, $62\frac{1}{2}$ per cent., 6·25, 0·625).
7. $\frac{1}{6}$ equals ($\frac{2}{3}$, $\frac{16\frac{2}{3}}{100}$, $16\frac{2}{3}$ per cent., ·164).
8. $\frac{7}{8}$ equals ($\frac{14}{16}$, ·88 −, ·87 +, 0·875, 87·5 per cent.).
9. $\frac{1}{3}$ is (greater than $\frac{1}{2}$, about 0·334, less than 35 per cent.).
10. $\frac{5}{12}$ is (less than $\frac{1}{2}$, about 2·2, very close to 42 per cent.).

When you pass Test 14 turn to Test 15, p. 170.

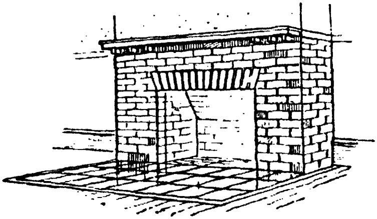
CHAPTER VI

HOW TO FIND THE AREAS OF COMMON FIGURES

59. Problems dealing with Area.—In a certain school some of the boys had garden plots. When Bob Burton entered the school most of the plots had been taken, but the teacher said: "Bob, we have two rectangular plots left. This one is five feet by seven, and that one is six feet by six. I think the soil is about the same. You may take your choice."

EXERCISES VIA

1. If you had been in Bob's place, and wished to make as much money as possible by raising vegetables, which plot would you have selected? Why?



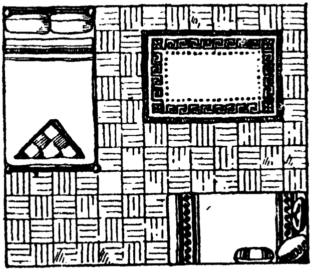
2. A floor in front of a fireplace is covered with square tiles of 1 foot side. How many tiles are used to cover a rectangular strip 8 ft. long and 3 ft. wide?

3. How many squares of turf, each 1 ft. side, will be needed to cover a rectangular front garden which is 10 ft. wide and 16 ft. long? Illustrate your solution of this problem by drawing, free-hand, a plan of the garden.

4 A builder wanted to buy enough square marble tiles, 6 in. side, to cover the floor of a rectangular room 10 ft. wide and 14 ft. long. How many tiles did he need?

60. Importance of measuring Area.—From the preceding exercises you have learned how important it is to know how to measure surfaces. We may buy carpets by the yard, but in order to know the number of yards required for a particular floor we must measure the floor. When we buy wallpaper we must know the area of the walls it is intended to cover. When we buy flowerpots for a window-sill we must measure the sill to see how many flowerpots it will hold. When a sailing master wishes to order a new sail of a triangular shape he must know how to calculate the number of square feet it will present to the wind when full set. In everyday life the areas of squares, rectangles, triangles, circles, and other figures are constantly being measured.

61. How to find the Area of a Rectangle.—One of the commonest and simplest figures is the rectangle. We shall now learn how people measure the amount of surface, or area, of a rectangular figure.



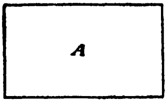
EXERCISES VIB

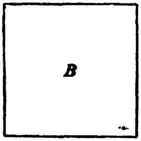
1. Here is the way the furniture in Jack's room is arranged. A line like this — stands for one foot. This □ stands for one square foot.

Say how many square feet are covered by the following pieces of furniture : (*a*) the rug ; (*b*) the couch ; (*c*) the bed.

2. Which covers the greater amount of floor surface, the bed or the couch ?

3. If the two rectangles, A and B, represented valuable pieces of land, each worth the same amount per square foot, which would you choose ?





4. In the preceding exercises we have been measuring *surface*. In each case our unit of measure was a *square foot*. Describe this unit. Name three other units which we can use to measure surface. Draw each of these.

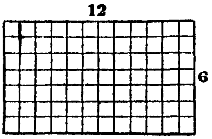
5. Is a 2-inch square the same as 2 square inches ? Explain clearly.

The number of square units in any surface is the *area* of that surface.

6. What is the area of a rectangle which is 12 in. long and 6 in. wide ?

Solution

Area $= 6 \times 12$ sq. in.
Hence area $= 72$ sq. in.



7. Write out, as in question 6, the solution for the area of each of the following rectangles :

(*a*) 10 ft. long and 8 ft. wide.
(*b*) 14 in. long and 9 in. wide.
(*c*) 20 yd. long and 18 yd. wide.
(*d*) 16 cm. long and 8 cm. wide.

8. The rule for the area of a rectangle is : The area of a rectangle is equal to the product of its base and height. The abbreviated rule, or *formula*, is

$$A = b \times h$$
$$\text{or } A = bh.$$

Which statement of the rule do you prefer? Why?

9. In the abbreviated rule or formula

$$A = bh$$

we must note that no sign is placed between b and h. Remember that bh means b times h. What is A, if $b = 10$ and $h = 7\frac{1}{2}$?

10. Find A (the number of square units in the area of a rectangle), if b (the number of units in the base) is $16\frac{1}{2}$ and h (the number of units in the height) is $8\frac{1}{2}$.

11. Complete the following table. Remember $A = bh$.

A	b	h	A	b	h
	10·5 in.	4·2 in.	40 sq. in.	8 in.	
	$5\frac{1}{3}$ ft.	$2\frac{1}{2}$ ft.		5 ft.	48 in.
	24·2 yd.	15·6 yd.		3 yd.	30 in.

12. Find the area of the picture on p. 9.

13. A garden, 32 ft. by 48 ft., contains a 2-ft. walk laid inside the garden along all four sides. The mid-points of the long sides are joined by a 2-ft. path. How much area is there for cultivation? Sketch the garden.

14. The boys in a school cemented a path 8 ft. wide and 24 ft. long. How much did they save for the school if the labour would have cost 3*s*. 6*d*. per square yard?

15. Find the cost of laying a cement path in front of your school at local prices.

16. What would it cost to cover your kitchen floor with linoleum at 5*s*. 6*d*. per square yard? Draw a plan of the floor, giving its dimensions.

17. How many sheets of paper 6 in. by 9 in. could you

cut from a rectangular piece 18 in. by 27 in.? Would there be any waste?

18. How many sheets of paper $8\frac{1}{2}$ in. by 11 in. can be cut from a rectangular piece 27 in. by 36 in.? Would there be any waste? If so, how many square inches? The waste is what per cent. of the original sheet?

19. Some contractors estimate the cost of building a one-story house by multiplying the number of square feet of floor space by a certain amount. At 15*s*. for each square foot of floor space, what will it cost to build a one-story house 32 ft. by 36 ft.?

20. How much will it cost, at 9*d*. per square foot, to stain and polish a table-top 28 in. by 60 in.?

21. An apple orchard contains 36 rows of trees, with 24 trees in a row. Last year the average yield was $1\frac{3}{4}$ bu. to the tree. What was the value of the crop at 6*s*. a bushel?

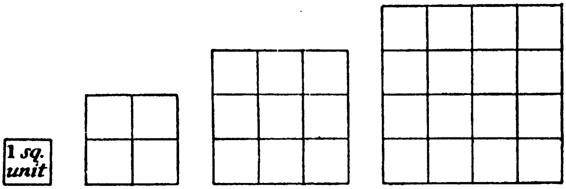
22. Make a drawing to represent a rectangular garden that is 125 ft. long and 60 ft. wide. Now show that the garden is enclosed by a walk that is 4 ft. wide. Find the number of square yards in the area of (*a*) the garden and (*b*) the walk.

23. Mrs Barker wishes to carpet a room 19 ft. by 17 ft. She is planning to buy carpet 30 in. wide. She sees that whether she lays the carpet lengthwise or crosswise in the room one strip must be turned under. How should she lay the carpet so as to have the least amount of waste (turned under)? She could not buy a part of a yard. How many square feet will be wasted? The waste is what per cent. of the amount purchased?

62. How to find the Area of a Square.—A square is a particular kind of rectangle. It is a rectangle with all sides equal. For this reason there is a special rule for finding the area of a square.

EXERCISES VIc

1. What is the area of each of the squares in the following figure? How many square units are there in the area of a square each side of which is 5 units? $10\frac{1}{2}$ units? 2·4 units?



2. Show that the abbreviated rule, or formula, for the area of the square may be written as $A = s \times s$, instead of $A = b \times h$.

3. We usually write the formula for the area of a square in this form:

$$A = s^2.$$

The letter s with a small 2 placed above and to the right of it means $s \times s$. In the same way 5^2 means 5×5, and b^2 means $b \times b$. What is A if $s = 5$ units? If $s = 10\frac{1}{2}$ units?

4. We read the formula $A = s^2$ as "A equals s squared." The figure 2 is called an index. It tells how many times the number is to be taken as a factor. For example, s^2 means $s \times s$; s^3 means $s \times s \times s$. What is the value of 5^2? of 4^3? of 2^5? of $(6·2)^2$? of $(10\frac{1}{2})^2$? of $(2·8)^2$?

5. What is s^2 if $s = 7$? if $s = 8\frac{1}{2}$? if $s = 4·2$?

6. Which of the following surfaces would you measure in square inches? in square feet or square yards? in square rods or acres? in square miles?

The floor of your classroom; this page; Ireland; a garden; a farm in Yorkshire; a ranch in Australia.

7. One square mile contains 640 acres. Complete the

following: $\frac{1}{2}$ sq. mile contains —— acres; $\frac{1}{4}$ sq. mile contains —— acres; $\frac{1}{8}$ sq. mile contains —— acres.

8. How many miles of fence will it take to enclose a square mile of land?

9. Mr Randolph has a square farm which contains 40 acres. Is it $\frac{1}{2}$ mile, $\frac{1}{4}$ mile, or $\frac{1}{8}$ mile on each side?

10. How many acres are there in a farm which is $\frac{1}{8}$ mile square? one that is $\frac{1}{2}$ mile square?

11. Mr Wright has a square farm of 160 acres. How long is one side of his farm?

12. (*a*) The children of a certain class had a garden which was 30 yd. by 18 yd. How many square feet did it contain?

(*b*) The paths took up 300 sq. ft. How much was left for gardening?

(*c*) There were 40 children in the class. How many square feet of garden space did each child have?

13. Mary Curtis said that the area of a rectangle is the "base times the height." Her brother Edwin said "the area is the length times the width." Were both correct?

14. Many people have not a clear mental picture of an acre. It will be helpful to your class if you stake out an acre on the school playing-field or on some other field.

15. If you wish to stake out a rectangular piece of ground containing an acre and make it 16 rods long, it must be —— rods wide; if 20 rods long it must be —— rods wide; if 40 rods long it must be —— rods wide. See Tables for Reference, p. 219.

16. A room is 24 ft. long, 16 ft. wide, and 10 ft. high. At 10*d.* a square yard what would be the cost of painting the walls, floor, and ceiling of the room? (Deduct 10 per cent. for windows.)

17. How many acres of land are there in a field that is $\frac{1}{2}$ mile long and $\frac{1}{4}$ mile wide?

18. Mr Builder intended having a porch 6 ft. by 9 ft.

Later he increased each dimension of the porch 25 per cent. How much did he increase the area ?

19. Complete the following table of squares :

2^2 equals 4
3^2 equals 9
4^2 equals 16
5^2 equals ——
6^2 equals ——
7^2 equals ——
8^2 equals ——
9^2 equals ——
1^2 equals ——
50^2 equals ——

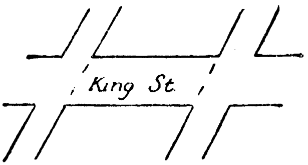
10^2 equals ——
11^2 equals ——
12^2 equals ——
13^2 equals ——
$1{\cdot}1^2$ equals ——
$1{\cdot}2^2$ equals ——
$1{\cdot}3^2$ equals ——
$1{\cdot}4^2$ equals ——
$2{\cdot}2^2$ equals ——
$2{\cdot}02^2$ equals ——

20^2 equals ——
30^2 equals ——
15^2 equals ——
$1{\cdot}5^2$ equals ——
25^2 equals ——
$2{\cdot}5^2$ equals ——
$0{\cdot}8^2$ equals ——
$0{\cdot}1^2$ equals ——
$0{\cdot}5^2$ equals ——
$2{\cdot}0^2$ equals ——

20. If the numbers with the index 2 in the table tell how long the sides of different squares are, then the corresponding numbers represent the —— of these squares.

21. How many bushels of potatoes are needed to plant a field 40 rods long and 20 rods wide, if 12 bushels are required to plant 1 acre ?

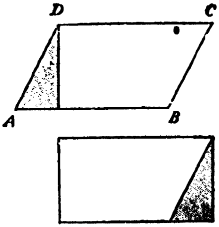
63. How to find the Area of a Parallelogram.—Mr Johnson, a borough surveyor, was preparing to pave King Street between two cross-streets. He saw that the street formed a parallelogram. It was necessary for him to find the area of a parallelogram. How do you think Mr Johnson solved this problem ?



An experiment to find the area of a parallelogram :

(*a*) Construct on cardboard a parallelogram, such as ABCD. From D draw a line to AB, making it perpendicular to DC.

Then cut off the shaded triangle, and place it as shown in the lower figure.



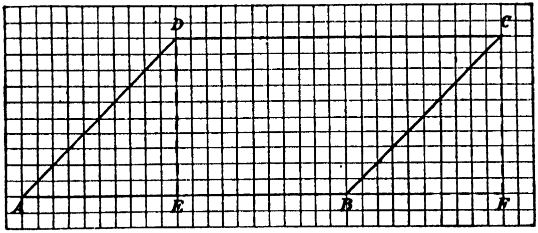
(*b*) The parallelogram has now been changed into a ——.

(*c*) Has the height (altitude) of the parallelogram been changed? Has the base been changed? Has the area been changed?

(*d*) What does this experiment show about the area of a parallelogram and the area of a rectangle having the same base and the same height?

A second experiment to find the area of a parallelogram:

(*a*) The parallelogram ABCD in the next figure has been drawn on squared paper. The base AB has been produced, and lines DE and CF, perpendicular to AB, have been drawn from D and C. Thus two triangles, ADE and BCF, have been formed.



(*b*) Estimate as accurately as you can the number of square units of area in the triangle ADE, in the triangle BCF. Compare your results with the results of the others in your class.

(*c*) What is true of the areas of the triangles ADE and BCF?

(*d*) Imagine that you cut off the triangle ADE and move it to BCF. Show that you get the rectangle CDEF, which is just as large as the parallelogram with which you started.

(*e*) What is true of the length of the base of the rectangle CDEF and the length of the base of the parallelogram ABCD?

Of the height (altitude) of the rectangle CDEF and the height of the parallelogram ?

(*f*) Write the formula for finding the area of the rectangle CDEF.

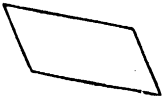
EXERCISES VID

1. State the rule for finding the area of a parallelogram. Show that the shortest way to express the rule for finding the area of a parallelogram is by using the formula $A = bh$.

2. What line would you measure to find the height of the preceding parallelogram ? Where else could the height be measured ?

3. These experiments illustrate the principle that the area of a parallelogram is always equal to the area of the rectangle which has the —— base and the same ——.

4. Explain clearly what the altitude of a parallelogram is. Draw the altitude of the parallelogram shown here. Find the area. Measure only those parts that you need.



5. The area of a parallelogram depends upon its —— and its ——. If the area of a parallelogram is 20 sq. in., and its altitude is very, very small, then its base is very, very ——.

6. Complete the following, using the formula $A = bh$:

(*a*) If b equals $6\frac{1}{2}$ in. and h equals 12·5 in., then A = —— sq. in.

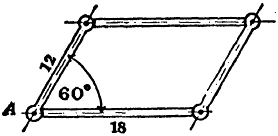
(*b*) If h equals $3\frac{1}{8}$ ft. and b equals $8\frac{1}{16}$ ft., then A = —— sq. ft.

(*c*) If h equals 16·2 units and b equals 27·43 units, then A = —— square units.

(*d*) If A equals 235 sq. in. and b equals 25 in., then h = —— in.

(*e*) If b equals 99·6 and h equals 5·1, then A equals nearly ——.

7. How will it affect the area of this parallelogram if the shape is changed so that the angle A becomes 90°? If angle A is changed to 30°? Illustrate this by a parallelogram made of four strips of cardboard pinned together.



8. Think of a parallelogram whose base is b and whose altitude is h. Now think of a parallelogram whose base is twice the base of the first parallelogram, and whose altitude is the same as that of the first one. How does the area of the second parallelogram compare with that of the first?

9. Think of a parallelogram. Now think of another with a base and an altitude each twice as great. Compare the areas of the two parallelograms.

10. In the formula $A = bh$, what change is made in A if b becomes twice as large as it was and h remains fixed? if both b and h become twice as large?

11. What is the value of A in the formula $A = bh$ when $b = 10$ and $h = 10{\cdot}1$?

12. Construct a parallelogram ABCD in which AB (the base) is 3 inches, angle A is 60°, and AD is 2 inches. What measurement would you need to make if you wanted to find the area of the parallelogram?

13. Measure the altitude of the figure which you drew for question 12, and find its area to the nearest tenth of a square unit. At this stage you may consider your work satisfactory if your result is anywhere between 5 and 5·4 sq. in. The best result is 5·2 sq. in.

14. If you had made angle A in question 12 an angle of 90°, with AB and AD remaining the same, what would have been the area of the figure?

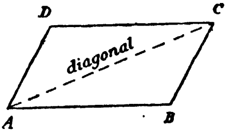
15. Mary's flower-bed was in the shape of a parallelogram. The sides were all the same length, 12 ft. The

altitude was 8 ft. Make a drawing to represent the garden and find its area.

16. Supply the missing numbers in the table. Use the formula $A = bh$.

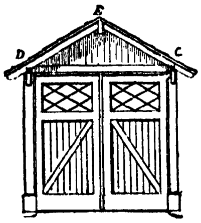
A	*b*	*h*	A	*b*	*h*
	$6\frac{3}{8}$	$3\frac{1}{2}$	56	7·8	
100	$8\frac{1}{4}$			1·6	$2\frac{1}{4}$
	$12\frac{1}{4}$	$16\frac{1}{8}$		20·8	$\frac{1}{2}$
	27	$4\frac{1}{8}$		4·45	$2\frac{1}{2}$

17. With a sharp knife cut a parallelogram along the diagonal into two triangles. Then show, by fitting one triangle on the other, that the two triangles are equal, and that one of the triangles equals —— of the parallelogram.



18. The ratio of the area of one of the triangles in question 17 to the area of the parallelogram is —— to ——.

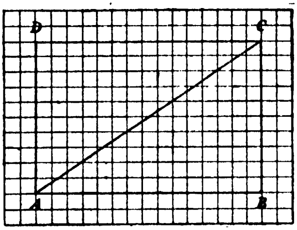
64. Finding the Area of a Triangle.—George and Jack wanted to find out how many square feet of tar-felt it would take to line the interior of a garage. An end view of the garage is shown here. They had difficulty in finding the area of the triangular part DCE. Do you know how George and Jack finally found how much paper was needed? The following experiments will help you.



EXERCISES VIE

1. An experiment to find the area of a triangle:

(*a*) Here is a rectangle drawn on squared paper; note that it is divided into two triangles, ABC and ACD. Estimate, as accurately as you can, the number of square units in triangle ABC, in triangle ACD.



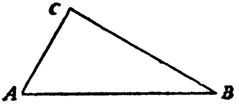
(*b*) How many square units are there in the rectangle ABCD?

(*c*) What is the relation between the area of the triangle ABC and the area of the rectangle ABCD?

(*d*) How could you find the area of the triangle if you knew the area of the rectangle?

2. A second experiment to find the area of a triangle:

(*a*) Draw an enlarged copy of triangle ABC. From C draw a line CE parallel to AB.

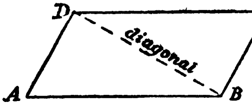


(*b*) Through B draw a line parallel to AC meeting CE at D. What kind of figure have you now? Does the figure which you drew look like the one for question 3 below?

(*c*) Are the two triangles equal? Test by cutting out the triangles and placing one over the other.

(*d*) What is the relation between the area of a triangle ABC and that of the parallelogram ABCD?

3. (*a*) The diagonal BD divides the parallelogram ABCD into two ——.



(*b*) The area of each triangle is —— of the area of the parallelogram.

(*c*) If the formula for the area of the parallelogram is $A = bh$, then the formula for the area of one of the triangles is $A = \frac{b \times h}{?}$.

4. Cut out two triangles exactly alike and show that they can be placed together to form a parallelogram.

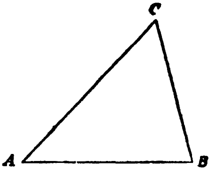
5. Show that the formula $A = \frac{bh}{2}$ is a good way to state the rule for finding the area of a triangle.

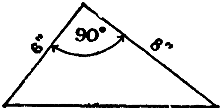
6. Find the areas of triangles that have the following bases and heights :

(*a*) b is $8\frac{1}{2}$ and h is 10·7 units.
(*b*) b is 10·45 and h is 6·3 units.
(*c*) b is 12·25 and h is 8·42 units.
(*d*) b is 6·5 and h is $2\frac{1}{2}$ units.

7. Draw a triangle in which b equals 4 in. and h equals 3 in. Have all of the triangles drawn by the different members of the class the same shape ? the same area ?

8. A teacher drew this triangle on the blackboard and asked her pupils to copy it and also to draw a dotted line showing its height, or altitude. Most of the pupils considered AB as the base. What height (or altitude) should they have drawn ? A few said that AC might be considered as the base. What line do you think they drew ? Show the line drawn by the pupils who took BC as the base.





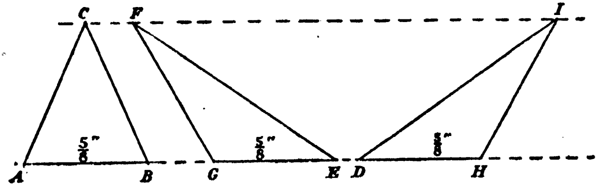
9. Find the area of the triangle shown in this figure.

10. Two triangles have equal bases; one has twice the height of the other. Compare their areas. Make a drawing to illustrate your method of making the comparison.

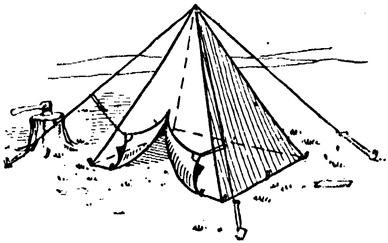
11. Compare the areas of the three triangles in the figure on p. 154. The line CI is parallel to the line AH.

12. What conclusion do you draw from question 11 ?

13. The area of a triangle is 40 sq. ft. Its base is 10 ft. Find the altitude.



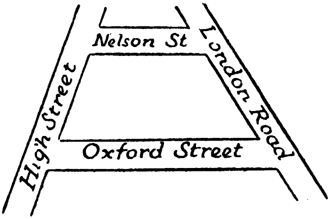
14. Find the areas of triangles whose dimensions are :



(*a*) $b=24$ in., $a=1$ ft.
(*b*) $a=4{\cdot}6$ ft., $b=18$ in.
(*c*) $a=30$ in., $b=\frac{1}{2}$ yd.
(*d*) $a=10$ cm., $b=10$ mm.
(*e*) $a=12{\cdot}4$ units, $b=8\frac{1}{2}$ units
( ) $a=16\frac{1}{2}$ ft., $b=2$ rd.

15. How many square yards of canvas are required to make a square tent like the one shown in this figure? The altitude of each side of the tent is 12 ft., and each side of the base is 10 ft.

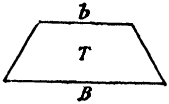
65. The Area of a Trapezium.—A *trapezium* is a figure having four straight sides, two of which are parallel. In the plan the plot of land which is bounded by High Street, Nelson Street, Oxford Street, and London Road is in shape a trapezium.



An experiment to find the area of a trapezium :

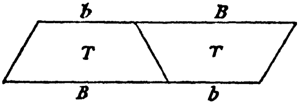
(*a*) Cut out two trapeziums, exactly alike. Letter each trapezium as shown in the figure here. Then place them together, as suggested by the next figure, and a —— will be formed.

(*b*) The area of one of the trapeziums is —— of the entire parallelogram.



(*c*) If B represents the lower base, and b the upper base of the trapezium T, what will represent the base of the parallelogram?

(*d*) Show that the area of the parallelogram is $h \times (B + b)$.



The brackets show that **B** and b are to be added before being multiplied by h.

(*e*) Show that the area of the trapezium T is represented by the formula $T = \frac{h \times (B + b)}{2}$.

Exercises VIf

1. Find the area of a trapezium T if $h = 10$ in., $B = 16$ in., and $b = 12$ in.

2. Find T if $h = 5$, $B = 8$, and $b = 6\frac{1}{2}$ units.

3. What is the area of the plot of land in the plan in § 65 above?

4. A master asked the boys in his class to write down the formula for finding the area of a trapezium. Here are some of the answers:

Henderson wrote $T = \frac{h + (B \times b)}{2}$.

Pringle wrote $T = \frac{h}{2}(B + b)$.

Marr wrote $T = h \times \frac{B + b}{2}$.

Davison wrote $T = \frac{hB}{2} + \frac{hb}{2}$.

Sanderson wrote $T = \frac{h(B + b)}{2}$.

Thomson wrote $T = \frac{hB + b}{2}$.

Which boys do you think gave the correct formula?

5. What result would each boy in question 4 obtain, if $h=10$, $B=16$, and $b=14$?

6. What special help should be given to each of the boys in question 4 who gave the formula incorrectly?

7. Supply the missing parts in this table. Use the formula $T=\frac{h\times(B+b)}{2}$.

T	h	B	b	T	h	B	b
	$10\frac{1}{2}$	$8\frac{1}{2}$	$6\frac{1}{2}$		2 ft.	20 in.	16 in.
	4·6	20	6·4		18 in.	2 ft.	$1\frac{1}{2}$ ft.
	3·8	12·2	8·3	100		10	10

66. How to find the Area of a Circle.—Follow carefully the experiment given here. You will then learn how to find the area of a circle.

(*a*) In Fig. 1 the side of the square is equal to the radius of the circle. What is the area of the square?

(*b*) Show that $A=r^2$ represents the area of one small square in Fig. 2.

(*c*) Show that the formula $A=4r^2$, represents the area of the large square formed by the four small squares in Fig. 2.

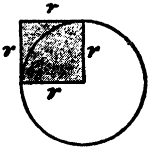


FIG. 1

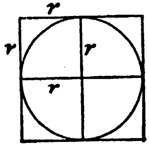


FIG. 2

(*d*) In Fig. 2 is the area enclosed by the circle as much as $4r^2$? Why? Do you think the area enclosed by the circle is more than r^2? More than $2r^2$? More than $3r^2$?

(*e*) In Fig. 3 the radius of the circle is 10 units. Point to the square which represents r^2.

(*f*) Count the number of the small squares in the circle, estimating fractional parts.

(*g*) Make a table of the results obtained by the others in your class, and find the average number of small squares.

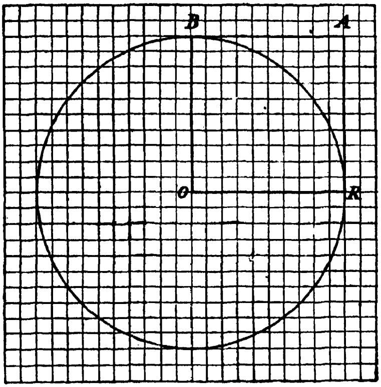


FIG. 3

(*h*) Since $r^2 = 100$ of the small units, how many r^2 are there in the area of the circle ? What conclusion do you draw from this experiment?

EXERCISES VIG

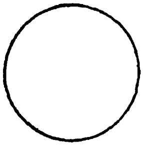
1. People who have measured very carefully the area of a circle agree that the area of any circle is very close to 3·14 times the square of the radius. The formula for the area of a circle, therefore, is $A = 3 \cdot 14\ r^2$ or

$$A = 3\tfrac{1}{7}\ r^2.$$

How close did the average result obtained by your class come to $3 \cdot 14\ r^2$?

2. Find A if $r = 4$ in. ; if $r = 8$ in. ; if $r =$ 1 in. ; if $r = \frac{1}{2}$ in.

3. Find the area of the circle shown here.



4. How much will it cost, at 1*s*. per square foot, to cement the bottom of a circular bathing pool whose radius is 20 ft. ?

5. Find the area of a circle whose diameter is 7 in.

6. If George is willing to pay 6*d*. for a jam tart the radius of which is 3 in., how much could he afford to pay for one just as thick with a radius of 6 in.?

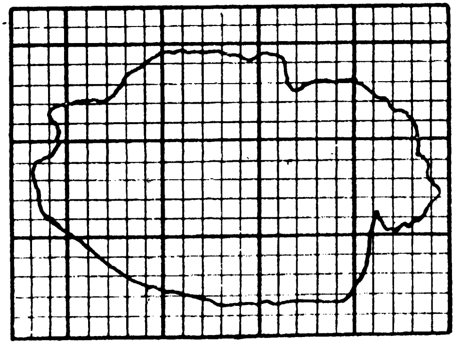
7. If the radius of one circle is twice as large as the radius of another circle, the area of the large circle will be —— times the area of the small circle.

8. If a circular piece of leather with a 4-in. radius is worth 2*s*., then a circular piece of the same material with an 8-in. radius is worth ——.

9. Suppose you had your choice of either a piece of gold foil in the shape of a circle whose circumference was 16 cm, or a piece of gold foil in the shape of a square whose perimeter was 16 cm. Which would you choose?

10. Find the area of a circular flower-bed the radius of which is 5 ft.

67. How to estimate Areas with Squared Paper.—In the accompanying figure the outline of the county of Norfolk has been traced on squared paper. The map from which the outline was taken was drawn on the scale of 30 miles to 1 inch. It is clear, therefore, that here 1 square inch will represent 900 square miles, and therefore 1 small square of the

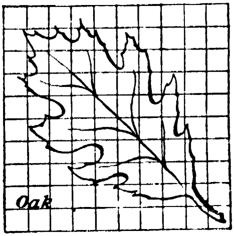


1 inch represents 30 miles

squared paper represents 9 square miles. By carefully counting the number of small squares we shall be able to obtain a reasonably correct answer to the question, What is the area of the county of Norfolk?

EXERCISES VIh

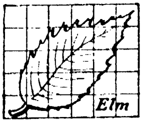
1. What is the area of the county of Norfolk? Give the answer in square miles.



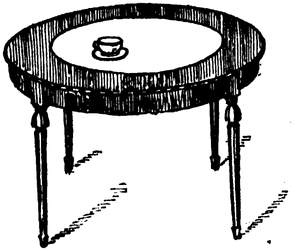
2. Find, by a method similar to that shown above, the area of the county of Leicester.

3. In a nature study class the boys and girls were asked to find out how many times larger an oak-leaf was than an elm-leaf. How could they do this?

4. The figures here show that Mary's oak-leaf was about —— times as large as the elm-leaf. Each leaf was actually four times as large as shown here.



68. How to find the Length of the Circumference of a Circle.—A lady wished to put a lace border round the circular tablecloth shown here. She saw that she would have to find the circumference of a circle (distance round it). The diameter of the circular tablecloth was 48 in. Do you know how to find the circumference of a circular object?



EXERCISES VII

1. Measure the diameter of some circular object, such as a plate, a bucket, or a half-crown, and record your result in a table like that below.

Now measure the circumference, and record that in the table.

Object measured	Diameter	Distance round, or Circumference	Circumference divided by Diameter
For question 1 . .			
For question 4 . .			

2. Divide the circumference by the diameter. Record the result.

3. Note that others in your class obtained different results for the diameters and the circumferences. Compare the results for "Circumference divided by Diameter."

4. Make a disk of heavy cardboard. Mark a point on the outer edge of the disk. Letter the point P. Place P at zero on a ruler and roll the disk along the stick until the point P again touches the stick. Measure the diameter of the disk and the length of the edge. Record your results in the table of question 1.

5. Very careful pupils have found, by the method used in question 4, that "Circumference divided by Diameter" equals approximately 3·14, or $3\frac{1}{7}$. If your answer lies between 3·05 and 3·25 your work is satisfactory.

6. Complete the following: The circumference of a circle is —— times its diameter.

7. If you know the diameter of a circle you can find its

circumference by —— by $3\frac{1}{7}$. If you know the circumference of a circle you can find its diameter by —— by $3\frac{1}{7}$.

The ratio of the circumference of a circle to its diameter is not exactly $3\frac{1}{7}$, nor is it exactly 3·14. Its value was once worked to more than 900 decimal places, but it is now known that the result would never "come out even." People generally use the symbol π to represent this ratio. For all practical purposes use $3\frac{1}{7}$ or 3·14 for π.

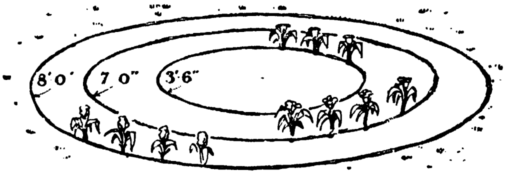
8. Show that the formula $C = \pi d$ is useful in finding the circumference of a circle.

9. Solve each of the following :

(*a*) If $d = 8$ then $C =$	(*d*) If $C = 50\frac{2}{7}$ then $d =$
(*b*) If $d = 4\frac{1}{2}$ then $C =$	(*e*) If $C = 314\frac{2}{7}$ then $d =$
(*c*) If $d = 12{\cdot}6$ then $C =$	(*f*) If $d = 25$ then $C =$

10. The circular running track at the Hillsdale School is 70 yards in diameter. In the mile run a boy must cover —— laps (times round).

11. George wished to make four flower-beds for the front garden, two for each side of the walk. George's sketch for one of the beds is shown here. George wished to plant each bed with two circular rows of tulips and a border of white hyacinths. George drove a stake at the centre of each plot, and used a string to mark out the circles. How many circles did he mark ? What radius did he take for each ?



CIRCLES WITH DIAMETERS AS SHOWN

12. What is the distance round the $3\frac{1}{2}$-ft. circle ? What is the circumference of each of the others ?

13. The tulips are to be set at intervals of about 7 in. How many tulips can George plant round the 3½-ft. circle? round the 7-ft. circle?

14. The hyacinths are to be set at intervals of about 8 in. How many hyacinth bulbs could be planted on the 8-ft. circle? What would George do if 8 were not contained a whole number of times in the circumference?

15. George found that tulips were 1*s*. 9*d*. per dozen and hyacinths 2*s*. 6*d*. What would it cost to buy enough plants for the four flower-beds?

EXERCISES VIJ

Some Arithmetical Problems based on the Cost of Carelessness

1. Mrs Newdriver forgot to put oil in her Ford when she set out for the town. As a result the motor got too hot, and the cylinders were ruined. She paid 15*s*. to be 'towed' to a garage, where the car was repaired. The charges were (*a*) 3½ hr. labour at 1*s*. 6*d*. an hour; (*b*) materials, £2 10*s*. When Mrs Newdriver paid the bill the mechanic said: " Madam, a quart of oil would have saved you all this time, trouble, and the £—— that you have had to spend."

2. James and Edgar each bought a new geography text-book. At the end of the year the boys sold their books. The dealer inspected them carefully and paid 1*s*. 2*d*. for James's book and 2*s*. for Edgar's. How do you account for the difference in the prices?

3. Mr Rose purchased a threshing machine for £840. The agent estimated that the value of the machine would depreciate, if reasonably good care were taken of it, as follows: 12½ per cent. of the original value the first year, 10 per cent. of the original value the second year, 8⅓ per cent. of the original value the third year, 5 per cent. of the original value the fourth year and each year thereafter

Mr Rose failed to take proper care of the machine, and consequently it was worth only £300 at the end of the fourth year. How much had his carelessness cost him in those four years?

EXERCISES VIk

Practice for Efficiency

1. Mr Smith, a business man, advised his employees to add two-figure numbers in this way:

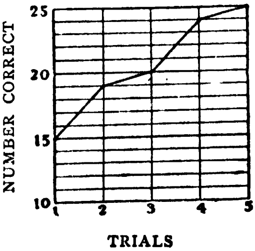
37
23 *Think*: $23 + 30 = 53$; $53 + 7 = 60$. Do all the work mentally.

Explain his method. Illustrate it by adding 26 and 32, 36 and 26, 48 and 18.

2. How many of the following additions can you do in 2 minutes, using the method shown above?

(*a*)	37	38	49	45	27	35	49
	24	26	34	28	38	18	28
(*b*)	46	44	53	48	37	18	69
	18	27	36	45	54	63	72
(*c*)	82	69	87	78	78	28	95
	65	76	39	49	87	97	18
(*d*)	97	87	87	87	77	76	85
	86	65	53	23	35	84	75

3. Grace kept this graphic record of her improvement in adding the two-figure numbers given in question 2. How many examples did Grace do on the first trial? on the second? on the fifth? Her score on the fifth trial was —— per cent. better than on the first trial. Can you do as well in 5 trials?



EXERCISES VII.

On the Interest Formula

1. We saw in the study of area problems that practical working rules are often written as formulas. This made the calculations easier. So, in interest problems, we can use a formula for solving nearly all the problems that we shall ever come across. Show that the interest formula is :

$$I=\frac{P.r.n}{100}.$$

2. State this formula in words.

3. You must remember in using this formula that P represents the sum of money (borrowed or lent), r the rate, and n the number of years. What does I represent ?

4. A man had to borrow £150 for 6 months at $4\frac{1}{2}$ per cent. How much interest did he have to pay ?

5. Find I if $P=£240$, $r=6$ per cent., and $n=2$ yrs.

6. What is I if $P=£720$, $r=8$ per cent., and $n=4$ yrs. ?

7. Use the formula to find the interest if $P=£800$, $r=5\frac{1}{2}$ per cent., and $n=4\frac{1}{2}$ yrs.

8. Suppose that you borrow £5 for 40 days at 6 per cent. How much interest will you pay ?

SOLUTION

$$I=5\times\frac{6}{100}\times\frac{40}{360}$$

Why $\frac{40}{360}$?

9. Find the interest on £12 for 50 days at 6 per cent.

10. What is the interest on £15 for 60 days at 6 per cent. ?

11. Find I when P is £180, $n=\frac{1}{4}$ yr., and $r=8$ per cent.

RUNNING THE MULTIPLICATION 'HURDLE' TEST

You should practise this test until you can 'take' these hurdles in 15 minutes. It is desirable that you practise once a week. More than one error on each hurdle knocks it down. You should finish without knocking a single hurdle.

Hurdle 1

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
9·38	8·96	431	739	805
12	23	102	302	46

Hurdle 2

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
6·24	·19	2·45	$7\frac{2}{5}$	12
4·2	·04	16	6	$8\frac{2}{3}$

Hurdle 3

(*a*)	(*b*)	(*c*)	(*d*)
$6\frac{2}{7} \times 14$	$9 \times 10\frac{1}{3}$	18·75	7·56
		175	$14\frac{3}{4}$

Hurdle 4

(*a*)	(*b*)	(*c*)	(*d*)
$8 \times 6\frac{3}{4}$	$9 \times 6\frac{1}{4}$	$\frac{3}{4}$ of $20\frac{1}{2}$	$\frac{2}{3}$ of 6·48

Hurdle 5

(*a*)	(*b*)	(*c*)	(*d*)
2 ft. 5 in.	8 lb. 6 oz.	$\frac{1}{2} \times \frac{1}{2}$	$4\frac{3}{4}$yd. $\times 1\frac{1}{2}$
×4	×5		

Revision Exercises VI

1. Keeping in mind the geometric figures and principles discussed in this chapter, you will recognize that some of these formulas are true; others are false. Make a list of the formulas that are true.

(*a*) $C=\pi d$
(*b*) $C=2\pi d$
(*c*) $C=2r\pi$
(*d*) $d=2r$
(*e*) $r=\frac{1}{2}d$
(*f*) $d=\frac{C}{\pi}$
(*g*) $\pi=\frac{C}{d}$
(*h*) $d=2\pi r$
(*i*) $A=\frac{bh}{2}$
(*j*) $A=bh$
(*k*) $A=\frac{(B+b)h}{2}$
(*l*) $A=2\pi r^2$

2. Say in words what each true formula stated above means.

3. Make a sentence which contains the following words: *the area of, surface, number of square units.*

4. Fred had 200 ft. of wire fence with which to enclose a plot of ground for a chicken run. He wondered what shape he ought to make the plot so as to have the run as large as possible. He used squared paper to find out. What do you think he did with the squared paper? Should he make the plot in the shape of a rectangle, a trapezium, a square, or a parallelogram?

5. Wild Rose, a Guernsey cow, yielded in one year 908 pounds of butter fat from 17,240 pounds of milk. What per cent. of her milk was butter fat?

6. Find the diameter, circumference, and area of a circle whose radius is one foot.

7. Name and describe some standard units of length, of area, and of volume.

8. Each part of Column A states something that belongs to some part of Column B. Match the parts in the two

columns by putting the right number in each square of Column B. Your score is the number of correct matchings.

COLUMN A	COLUMN B
(*a*) The circumference of a circle divided by the diameter.	☐ An index.
(*b*) The formula for the area of a square.	☐ Altitude of triangle.
(*c*) The formula for the area of a rectangle.	☐ $A = \frac{1}{2}bh$.
(*d*) A number which tells how many times some number is to be used as a factor.	☐ $A = \pi r^2$.
(*e*) A square mile of land.	☐ $3\frac{1}{7}$.
(*f*) The perpendicular drawn from any vertex to the opposite side of a triangle.	☐ Circumference.
(*g*) The formula for the area of a triangle.	☐ $A = bh$.
(*h*) The formula for the area of a trapezium.	☐ $C = \pi D$.
(*i*) The formula for the circumference of a circle.	☐ $2a + 2b$.
(*j*) The formula for the area of a circle.	☐ $A = s^2$.
(*k*) A word meaning the perimeter of a circle.	☐ $6x^2$.
(*l*) The perimeter of a rectangle.	☐ 640 acres.
(*m*) The surface of a cube.	☐ $A = \frac{1}{2}h(B + b)$.

Pupils may practise this test more than once by placing a narrow strip of paper over the squares and writing the letters thereon.

EXERCISES VIM

Just for Fun

1. *Stranger.* " What is the price of this pair of shoes ? "

Country shopkeeper. "If you want this pair of shoes I'll make you a real bargain. They have been in stock for some time. They cost me 25*s.* ; take them for 20*s.*"

Stranger. "A bargain!" (Hands the shopkeeper a £5 note ; the shopkeeper can't give change ; goes to his friend, the postmaster, to get change for the £5 note. Returns and gives the stranger the shoes and £4.) " I think I shall be pleased with this purchase." (Stranger leaves town.)

Enter postmaster, exclaiming : " This £5 note is counterfeit. You'll have to make it good."

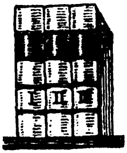
Shopkeeper. " Don't get excited. I'll see that you don't lose anything. This experience has been expensive for me."

What was the shopkeeper's loss ?

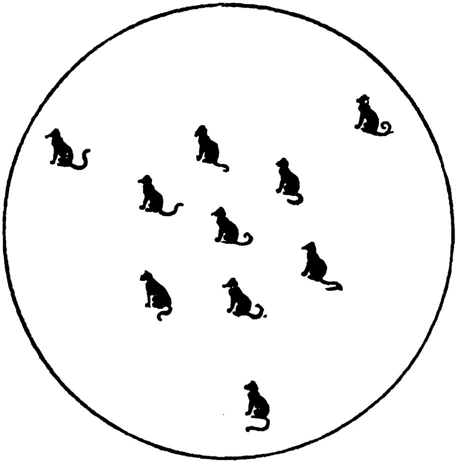
1	9	2
3	8	4
5	7	6

2. The nine digits are here arranged in a square so that the number in the second row (384) is twice the number in the first row, and the number in the bottom row three times that in the top row. There are three other ways of arranging the digits so as to produce the same result. Can you find them ?

3. " Do you know the bookworm problem ? " asked Professor Wise. " It's like this," he said. " A hard-working bookworm has actually bored a hole straight from the first page of Volume I to the last page of Volume III. (The three volumes are placed on a shelf in the regular order.) The leaves are altogether 2 in. thick in each volume and every cover is exactly one-fourth of an inch thick. How long a tunnel did the industrious worm bore in preparing his new tube ? "



4. It is possible to fold a piece of paper so that with one clip of the scissors a five-pointed star is cut. Of course, you may do it after a little experimenting—and again you may not!



5. A wizard is said to have placed ten cats inside a magic circle, as shown in the illustration, and hypnotized them so that they should remain stationary during his pleasure. He then proposed to draw three circles inside the large circle, so that no cat could approach another cat without crossing a magic circle. Try to draw the three circles so that every cat has its own space and cannot reach another cat without crossing a circle.

PRACTICE FOR ACCURACY AND SPEED

Test 15. Valuable Helps in Adding Fractions

In adding, I think of

1. *Halves* and *thirds* as —— *ths.*
2. *Halves* and *fifths* as —— *ths.*
3. *Halves* and *eighths* as —— *ths.*
4. *Halves* and *twelfths* as —— *ths.*
5. *Thirds* and *fifths* as —— *ths.*
6. *Thirds* and *eighths* as —— *ths.*
7. *Thirds* and *twelfths* as —— *ths.*

8. *Fourths* and *fifths* as —— *ths.*
9. *Fourths* and *eighths* as —— *ths.*
10. *Fourths* and *twelfths* as —— *ths.*
11. *Fifths* and *fifths* as —— *ths.*
12. *Fifths* and *eighths* as —— *ths.*
13. *Sixths* and *tenths* as —— *ths.*
14. *Eighths* and *twelfths* as —— *ths.*
15. *Thirds* and *thirds* as —— *rds.*
16. *Thirds* and *fourths* as —— *ths.*
17. *Halves*, *thirds*, and *sixths* as —— *ths.*
18. *Halves*, *thirds*, and *eighths* as —— *ths.*
19. *Halves*, *thirds*, and *twelfths* as —— *ths.*
20. *Thirds*, *fourths*, and *sixths* as —— *ths.*
21. *Thirds*, *fourths*, and *twelfths* as —— *ths.*
22. *Fourths*, *fifths*, and *tenths* as —— *ths.*
23. *Sixths*, *eighths*, and *twelfths* as —— *ths.*
24. *Halves*, *fourths*, and *fifths* as —— *ths.*
25. *Halves*, *fourths*, and *sixths* as —— *ths.*
26. *Halves*, *fourths*, and *eighths* as —— *ths.*
27. *Halves*, *fourths*, and *tenths* as —— *ths.*
28. *Halves*, *fourths*, and *twelfths* as —— *ths.*
29. *Halves*, *fourths*, and *sixteenths* as —— *ths.*
30. *Halves*, *fifths*, and *tenths* as —— *ths.*
31. *Halves*, *sixths*, and *eighths* as —— *ths.*
32. *Halves*, *sixths*, and *twelfths* as —— *ths.*
33. *Halves*, *thirds*, *fourths*, and *sixths* as —— *ths.*
34. *Halves*, *fourths*, *eighths*, and *sixteenths* as —— *ths.*

Test 16. Addition of Fractions

$\frac{1}{2}+\frac{3}{4}=$ $\frac{1}{8}+\frac{5}{8}=$ $\frac{3}{8}+\frac{11}{16}=$ $\frac{1}{2}+\frac{2}{3}=$

$\frac{1}{2}+\frac{11}{16}=$ $\frac{1}{10}+\frac{3}{5}=$ $\frac{3}{16}+\frac{15}{16}=$ $\frac{1}{2}+\frac{9}{16}=$

$\frac{1}{3}+\frac{5}{6}=$ $\frac{1}{12}+\frac{7}{12}=$ $\frac{5}{8}+\frac{5}{16}=$ $\frac{2}{3}+\frac{1}{4}=$

$\frac{1}{4}+\frac{3}{4}=$ $\frac{2}{3}+\frac{3}{5}=$ $\frac{5}{16}+\frac{7}{16}=$ $\frac{1}{4}+\frac{9}{16}=$

$\frac{1}{4}+\frac{11}{16}=$ $\frac{2}{5}+\frac{4}{5}=$ $\frac{7}{12}+\frac{7}{12}=$ $\frac{1}{10}+\frac{2}{5}=$

$\frac{1}{5}+\frac{7}{10}=$ $\frac{3}{4}+\frac{7}{10}=$ $\frac{13}{16}+\frac{15}{16}=$ $\frac{3}{8}+\frac{11}{12}=$

Test 17. Subtraction of Fractions

$\frac{1}{2}-\frac{3}{10}=$ $\frac{4}{5}-\frac{1}{5}=$ $\frac{7}{12}-\frac{1}{12}=$ $\frac{2}{3}-\frac{1}{2}=$

$\frac{1}{4}-\frac{1}{10}=$ $\frac{5}{6}-\frac{1}{5}=$ $\frac{9}{10}-\frac{1}{10}=$ $\frac{2}{5}-\frac{3}{10}=$

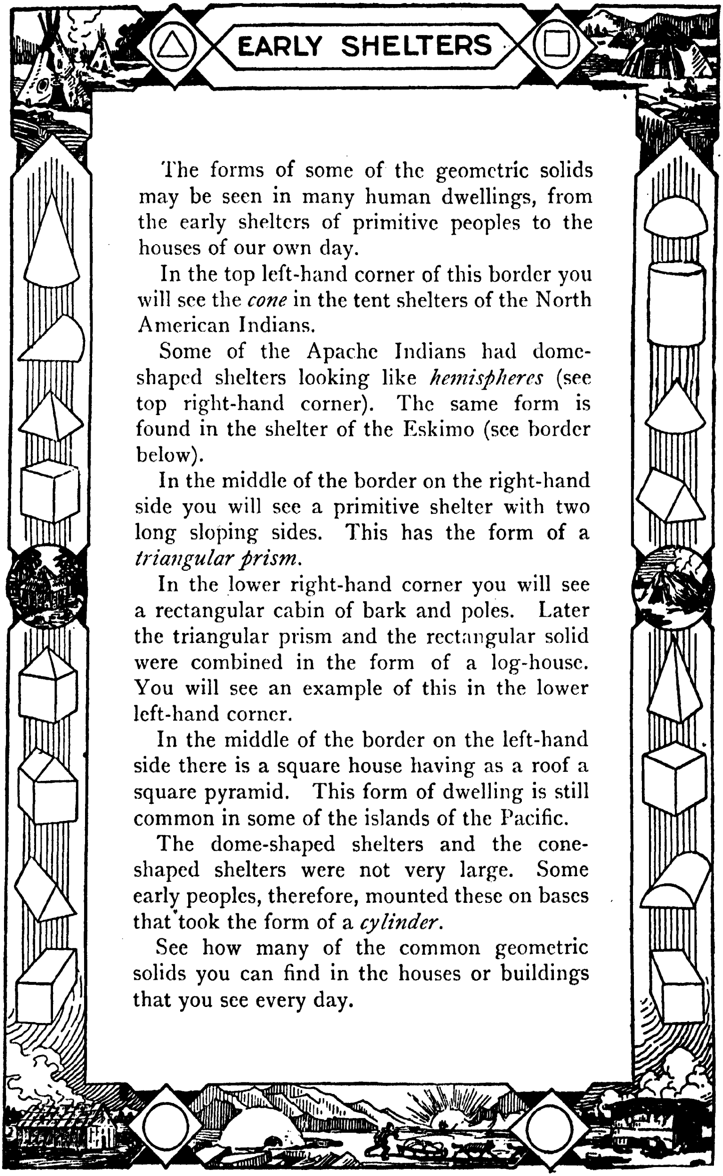
$\frac{2}{3}-\frac{1}{4}=$ $\frac{5}{6}-\frac{7}{12}=$ $\frac{9}{16}-\frac{1}{16}=$ $\frac{3}{4}-\frac{3}{16}=$

$\frac{2}{5}-\frac{1}{5}=$ $\frac{5}{8}-\frac{5}{12}=$ $\frac{11}{12}-\frac{5}{8}=$ $\frac{3}{10}-\frac{1}{4}=$

$\frac{3}{4}-\frac{2}{3}=$ $\frac{7}{8}-\frac{1}{2}=$ $\frac{13}{16}-\frac{1}{4}=$ $\frac{4}{5}-\frac{2}{5}=$

$\frac{3}{5}-\frac{1}{3}=$ $\frac{7}{8}-\frac{3}{16}=$ $\frac{15}{16}-\frac{3}{4}=$ $\frac{5}{6}-\frac{2}{3}=$

When you pass Test 17 turn to Test 18, p. 198.



EARLY SHELTERS

The forms of some of the geometric solids may be seen in many human dwellings, from the early shelters of primitive peoples to the houses of our own day.

In the top left-hand corner of this border you will see the *cone* in the tent shelters of the North American Indians.

Some of the Apache Indians had dome-shaped shelters looking like *hemispheres* (see top right-hand corner). The same form is found in the shelter of the Eskimo (see border below).

In the middle of the border on the right-hand side you will see a primitive shelter with two long sloping sides. This has the form of a *triangular prism.*

In the lower right-hand corner you will see a rectangular cabin of bark and poles. Later the triangular prism and the rectangular solid were combined in the form of a log-house. You will see an example of this in the lower left-hand corner.

In the middle of the border on the left-hand side there is a square house having as a roof a square pyramid. This form of dwelling is still common in some of the islands of the Pacific.

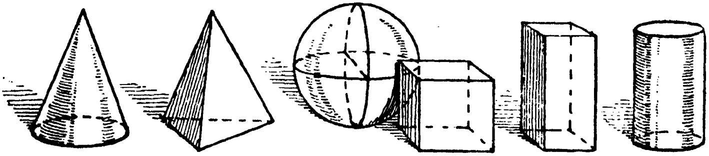
The dome-shaped shelters and the cone-shaped shelters were not very large. Some early peoples, therefore, mounted these on bases that took the form of a *cylinder.*

See how many of the common geometric solids you can find in the houses or buildings that you see every day.

CHAPTER VII

HOW TO MEASURE VOLUMES

69. How to measure the Volume of Common Solids.—In the picture below you see some of the common solids. All of these are shown in the border of the preceding page.



EXERCISES VIIA

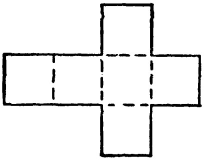
1. Look at the border showing early shelters, and find where each of the solids shown above is used.

2. See how many of these solids you can find in your classroom.

3. In the last chapter we studied surfaces. Surfaces have two dimensions, length and width. Which surface do you think of when you look at the cube? at the sphere? the rectangular prism? the triangular pyramid? the cylinder?

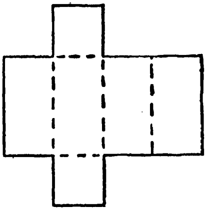
4. How many faces has a cube? What kind of figures are they? How many edges does a cube have? How are these edges formed? How many corners has it?

5. This figure shows you how to make a cube out of cardboard. Make a similar figure on a larger scale. Cut out the figure along the heavy lines; fold along the dotted lines. Join the edges by means of gummed paper. Try to make a cardboard cube at home.



6 Find the picture of a rectangular solid on p. 173. Any solid which has six sides, each side a rectangle, is called a *rectangular prism*. How is the cube like this solid? In what way is the cube a special form of the rectangular prism?

7. A rectangular solid may be constructed from cardboard cut like this figure. Construct a model at home and bring it to school so that you can show it to your teacher and compare it with the models made by others in your class. By means of a diagram or model explain the manner in which the surface area of a rectangular prism may be calculated.



8. Do you see any parallel lines in a rectangular prism? Where? Show that a rectangular solid contains perpendicular lines. Point to some right angles in a rectangular prism.

9. How many square feet of cardboard are needed to cover a box (a rectangular prism) that is 28 in. long, 20 in. wide, and $\frac{1}{2}$ ft. deep?

10. What is the surface area of a rectangular prism that is 18 in. long, 8 in. wide, and 6 in. high?

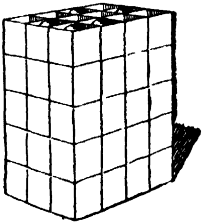
11. How does a solid differ from a two-dimension figure?

We learn from these exercises that a solid has three dimensions. It cannot lie in a flat surface (in a plane). In geometry we study only the form, size, and position of solids. This chapter deals chiefly with their size, or volume.

70. How to find the Volume of a Rectangular Prism.—Every week Mrs Giles sends a small crate of eggs to her daughter Kathleen in London. The picture below represents the egg crate. Each little cubic space holds one egg.

EXERCISES VIIB

1. How many units in the length of the egg crate? How many units in the width? How many eggs could Mrs Giles place in the bottom layer? How many layers did she have to fill? How many eggs does the crate hold?



2. How many would it hold if it were two units high? Three units high?

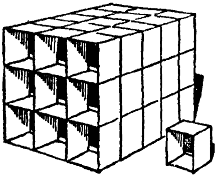
3. Give a rule for telling, without counting, how many eggs the crate would hold.

4. How many dozen eggs can be placed in a crate that is 6 units long, 6 units wide, and 6 units high?

Mrs Giles's egg-crate problem suggests how people measure the amount that a rectangular solid will hold. The measurement of volumes is often necessary. For example, Mr Giles needs to know the volume of his water-tanks, his haystacks, his corn-bins, and the like.

In measuring volume we need a standard unit of volume, just as we needed a unit of area in measuring surface and a unit of length for measuring straight lines. To measure the volume of a solid we compare it with a cube each of whose edges is a standard unit of length. Commonly used units for measuring volume are the cubic inch, the cubic foot, the cubic yard, and the cubic centimetre. Describe each of these units.

5. Find the "unit cube" in this figure. How long is it? how high? how wide? Why is it called a *unit of volume*?

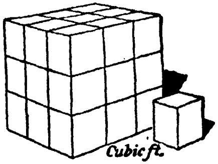


6. How many cubic units of volume are there in the bottom layer? in two layers? in the box?

7. Give a rule for finding the number of cubic units of volume in the box of the preceding figure without counting them.

8. Name something that is bought and sold by the cubic foot; by the cubic yard.

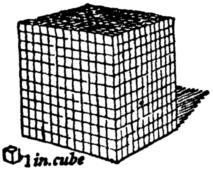
9. The smaller solid in this figure represents a cubic foot. What does the larger solid represent?



10. Complete: One cubic yard equals —— cubic feet.

11. What does the next figure show?

12. Complete: One cubic foot equals —— cubic inches.



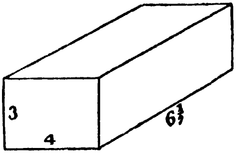
13. How many cubic inches are there in a chalk-box 4″ by 10″ by 6″? (4″ means 4 inches.)

14. Which will hold the greater amount of coal, a bin 6′ by $4\frac{1}{2}$′ by 8′ or a bin 7′ by $6\frac{1}{2}$′ by 5′? (6′ means 6 feet.)

15. Complete: To find the volume of a rectangular prism we find the product of the —— dimensions.

16. Find the volume, in cubic inches, of a rectangular solid that is 1 ft. long, 10 in. wide, and 8 in. deep.

71. The Formula for the Volume of a Rectangular Prism.—The volume of a rectangular prism is found by multiplying together the length, breadth, and height. The volume of the prism here shown is, therefore, 78 cubic units. Writing the above rule as a formula, we may say $V = lbh$.



EXERCISES VIIC

1. Find the volumes of rectangular solids of the following dimensions :

(*a*) 2″, 5″, 8½″.
(*b*) 11 cm., 3 cm., 5·2 cm.
(*c*) 8′, 4′, 3¼′.
(*d*) 2 rd., 5½ rd., 3 rd.
(*e*) 10 m., 30 m., 35 m.
(*f*) 4·5″, 2″, 3·3″.
(*g*) 8 ft., 6 ft., $\frac{1}{16}$ ft.
(*h*) 2 yd., 5 yd. ¼ yd.

2. Ground is to be excavated for a cellar 20′ by 18′ by 9′. How much will this cost at 4*s*. a load? A load is equal to about 1 cubic yard.

3. How many bushels of wheat can be loaded into a section of the hold of a ship, if the section be 35′ long and 8½′ wide and be filled to a depth of 6′? Allow 0·8 bu. for a cubic foot.

4. A coal-house, 12 ft. long and 8½ ft. wide, is filled to a depth of 5½ ft. How many tons of coal are there in it? Allow 90 lb. of coal to the cubic foot. (2240 lb. = 1 ton.)

5. The swimming tank at a certain school is 20′ by 65′. Its average depth is 4¾′. How much water will it contain? Allow 6¼ gal. for each cubic foot.

6. How many pounds does the ice-man lift when he delivers a block of ice 10 in. thick, 18 in. long, and 12 in. wide? Use 56¼ lb. as the weight of a cubic foot of ice.

7. A cubic foot of gold is 19·2 times as heavy as a cubic

foot of water. Would you accept, as a present, a block of gold 9″ by 9″ by 9″ if you had to agree to carry it home? (A cubic foot of water weighs 62·4 lb.)

8. Find the value of V in the formula $V = lbh$, if

(*a*) b equals $2\frac{1}{4}$ in., l equals $2\frac{1}{4}$ in., and h equals 10 in.
(*b*) b equals 3·2 in., l equals 2·4 in., and h equals 6 in.
(*c*) b equals 8·2 cm., l equals $6\frac{1}{2}$ cm., and h equals 10 cm.
(*d*) b equals 5·2 yd., l equals 2·05 yd., and h equals 3·1 yd.

9. David Richards said the formula for the volume of a rectangular prism might be written $V = bh$, in which b is the area of the base. Was he correct?

10. A quarter of an inch of rain fell on a 40′ by 60′ roof that was almost level. It ran into a cistern built in the form of a rectangular prism with a bottom 10′ by 10′. How much did the rain raise the level of the water?

11. What should be the depth of a rectangular box that is to hold 25 bu. if it is 4 ft. square? A bushel contains approximately 2150 cu. in.

12. A class in the Kingsley School made an aquarium that was 2 ft. long, 18 in wide, and $1\frac{1}{4}$ ft. deep. How many gallons of water were needed to fill it two-thirds full? Allow $6\frac{1}{4}$ gal. for each cubic foot.

72. Formula for the Volume of a Cube. — The volume of a cube may be found by means of a simple formula. The cube is a special form of the rectangular prism in the sense that its length, breadth, and height are all equal. Hence, if s represents the length of the edge of a cube we may express its volume by the formula $V = s \times s \times s$.

EXERCISES VIID

1. Say this in words: $V = s \times s \times s$.
2. Show that $V = s \times s \times s$ may be written as $V = s^3$.

This is read " V equals *s* cubed " or " V equals the cube of *s*."

3. Find the volume of a cube whose edge is 2 in. ; 5 yd. ; $\frac{1}{2}$ in. ; 2 cm. ; 3 yd. ; $1\frac{3}{4}$ in. ; 2·2 rd. ; 1 m. ; 0·1 m.

4. What is the index in the number 2^3 ? What does it mean ? Find 2^3, 3^3, 4^3, 5^3, $(\frac{1}{2})^3$, $(\frac{1}{3})^3$, $(\frac{1}{4})^3$.

5. Find the volume, in cubic centimetres, of the cube that you constructed from cardboard.

6. A standard bushel contains 2150·42 cu. in. How many cubic feet are there in one bushel ? Your result should be correct to hundredths.

7. A wheat bin, 6 ft. wide, 14 ft. long, and 4 ft. high, holds —— bu.

8. How many gallons in a cubical tank of 6 ft. side ? (1 cu. ft. = $6\frac{1}{4}$ gal.)

9. Find the volumes of cubes of the following dimensions :

(*a*) 2 in. (*c*) 0·2 in. (*e*) 3 ft. (*g*) 8·4 ft.
(*b*) 2 cm. (*d*) $\frac{1}{2}$ in. (*f*) 6 ft. (*h*) 50 units.

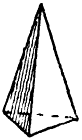
10. How should two rectangular solids, each 4″ by 6″ by 8″, be placed together to expose the least surface ?

73. Measuring the Volume of Pyramids.—In this picture you have an illustration of a geometric sol

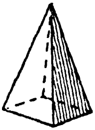
—viz., the pyramid. The story of the building of these great Egyptian pyramids is full of romance and interest.

All the faces of a pyramid are triangles, and all of them meet in a common point, the vertex.

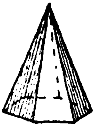
Pyramids are named according to the number of sides in their bases, as shown in the figures above.



TRIANGULAR PYRAMID



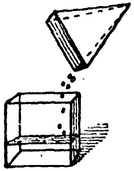
SQUARE PYRAMID



HEXAGONAL PYRAMID

Here is an experiment to find the volume of a pyramid:

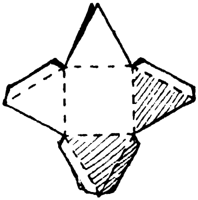
(*a*) Make a rectangular pyramid and a rectangular prism with equal bases and equal altitudes. Fill the pyramid with rice, water, or sand, and pour the contents into the prism. Repeat the process until the prism is full. What does this experiment show about the relative size of a pyramid and a prism with the same base and altitude?



(*b*) If performed accurately (*a*) shows that

id is one-third the

2. A certain pyramid has a square base, 30 in. side. Its altitude is also 30 in. What is its volume? What would its volume be if each side of the base were 60 in.?

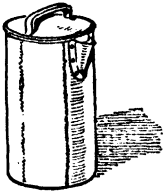


3. The area of the base of a triangular pyramid is 42 sq. in.; its altitude is $18\frac{1}{2}$ in. What is its volume?

4. Which is larger, a piece of cake in the form of a rectangular prism 2″ by 1″ by 1·5″ or one in the form of a rectangular pyramid whose base is 2″ by 3″, and whose height is 1·5″?

5. The pyramid of Cheops in Egypt has a base 764 ft. square and an altitude of 480 ft. Does it cover more or less than an acre of ground? About how many cubic yards of stone does it contain?

74. How to find the Volume of a Cylinder.—Mr Robson, the tinsmith, took an order to make a petrol can which would hold exactly 3 gallons. He wanted the radius of the base to be 3·5 inches. In order to find how high to make the can he had to know how to find the volume of a cylinder. The questions below will help you to learn how to find the volume of a cylinder.

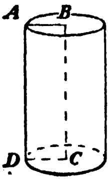


Exercises VIIf

1. A cylinder is formed by rotating a rectangle about one of its sides as an axis. How was the cylinder in this figure formed?

2. The radius of the base of this cylinder is 5 inches.

How many square inches are there in the area of the base? (Take $\pi = 3\frac{1}{7}$.)

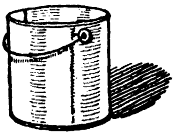


3. If you could fit cubic inches (inch cubes) into the bottom of this cylinder, how many could you get into the bottom layer? (Assume that the unit may be divided.)

4. If the height of the cylinder is 18 inches how many layers of inch cubes could be placed in the cylinder?

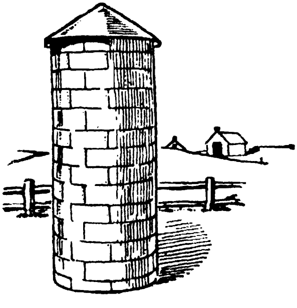
5. What is the volume of this cylinder—that is, how many cubic inches could be placed in the cylinder?

6. If the radius of the base of a cylinder is r inches, show that $3\frac{1}{7}\ r^2$ cubic inches could be placed in the bottom layer.



7. If the height of the cylinder is h inches, show that the volume is $3\frac{1}{7}\ r^2h$ cubic inches.

8. If r is the radius of the base, and h the height, show that the formula for the volume of a cylinder is $V = 3\frac{1}{7}\ r^2h$. Show that this formula may be written as $V = \pi r^2h$.



9. Find V if $r = 8$ in. and $h =$ 10 in.

10. Find V if $r = 6$ in. and $h =$ 1 ft.

11. How many cubic inches are there in a cylindrical bucket the radius of whose base is 4 in. and whose height is 7 in.?

12. Many objects are made in the form of a cylinder. Water-pipes, oil-tanks, boilers, and cisterns are usually cylindrical in shape. See if you can bring to class a list of five objects (seen at home, in the shop, on the farm, or in the factory) that are good examples of cylinders.

13. A farmer has a cylindrical silo (*i.e.*, an airtight chamber for storing fodder) which is 10 ft. in diameter and 25 ft. high. How many cubic feet will it hold ?

14. If the silo had a 5-ft. diameter and was 25 ft. high how much would it hold ?

15. The radius of a cylinder is 4 in. ; its volume is 301 cu. in. Find its height.

16. The petrol can in the problem at the beginning of this section was to hold 3 gal., or 832 cu. in. The radius of its base was 3·5 in. Can you now find how high it had to be made ? (Use $\pi = 3\frac{1}{7}$.)

17. What do you have to know to find the volume of a cylinder ? Of a rectangular solid ?

18. William wanted to make a cylindrical cup which would hold a quart. He knew that its volume must be 69·36 cu. in. He made the diameter 4 in. and the height 5·6 in. Did it hold a quart ? ($\pi = \frac{22}{7}$.)

75. How to find the Surface of a Cylinder.—In making the petrol can Mr Robson, the tinsmith, was especially interested in finding out how much tin to use. He had to find the total surface of the can—that is, the top and bottom and the curved surface. How many separate parts are there in such a cylindrical surface ? How do you find the area of the top and bottom of the can ?

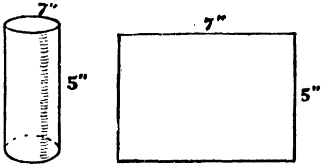
Since you have already learned how to find the area of the circular parts, we need only make a special study of the curved surface of the can.

EXERCISES VIIG

1. Cut out a piece of paper so that it will just cover the curved surface of a cylinder. What is the shape of the

paper when you lay it out flat? How can you find the area of the paper?

2. Look at the cylindrical piece of tin shown in this figure. Its circumference is 7 in. It has been split open and flattened out. What shape does it take? What are its dimensions? What is its area?



3. What are the dimensions of a piece of paper that will exactly cover the curved surface of a cylinder which has a circumference of 20 in. and whose height is 12 in.? What is the area of the paper? What is the area of the curved surface?

4. A tin cup is $3\frac{1}{2}$ in. high and 8 in. round. How many square inches of tin were used for the curved surface?

5. (*a*) How much tin would be used for the curved surface of a cylinder whose height is 18 in. and whose circumference is 22 in.? (Neglect the part used for soldering.) What bearing does this exercise have on Mr Robson's problem referred to at the beginning of this section?

(*b*) How much tin is needed for the top of the can? for the bottom?

(*c*) What is the total amount of tin needed?

6. A painter wishes to find the cost of painting a cylindrical water-tower whose diameter is 15 ft. and whose height is 84 ft. What would it cost at 10*d.* per square yard?

7. How many square feet of sheet iron would you need to make a stove-pipe 3 ft. long and 8 in. in diameter, allowing 1 in. for overlapping?

8. The pupils in one school measured the dimensions of a large cylindrical tank built by an oil company for storing petrol. The circumference was 78·54 ft. and the height was 20 ft. What problems do you think the pupils

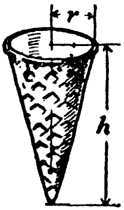
wanted to solve? State two problems that you could solve by using these facts.

9. Mr Farmer offered his son John 9*d.* a square yard to paint the surface of their silo. It was 20 ft. high and 14 ft. in diameter. John reckoned that he could complete the task in 4 days, thereby earning —— a day.

10. Find the entire surface of cylinders whose dimensions are as follows:

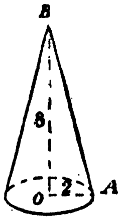
(*a*) $r=4$ cm., $h=6$ cm.
(*b*) $r=8{\cdot}4$ cm., $h=4{\cdot}2$ cm.
(*c*) $r=1$ ft., $h=6$ in.
(*d*) $r=8$ in., $h=\frac{3}{4}$ ft.

76. Measuring the Volume of a Cone.—Some of the girls of Oldtown School decided to sell ice-cream cones at the school garden-party. Each cone was 4 in. high, and the base of each had a radius of 1 in. They wanted to know how many cones could be filled with a gallon (277 cu. in.) of ice-cream. How could they find out? These exercises will show you how to find the *volume* of a cone.



EXERCISES VIIH

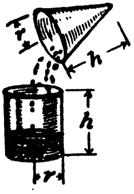
1. A cone is formed by rotating a right-angled triangle about one of its sides. Show how the cone represented in this figure is formed.



2. What is the radius of the cone shown here? The height?

3. Try to get a cylinder and a cone with the same radius and the same height. (See the next figure.) Fill the cone with rice, water, or sand, and pour it into the cylinder. Repeat the process until the cylinder is full. The cylinder is how many times as large as the cone?

4. This experiment, if performed accurately, shows that the volume of a cone is one-third of the volume of a cylinder with the same base and the same altitude. Show that the formula is $=V\frac{3\frac{1}{7}\times r^2\times h}{3}$.



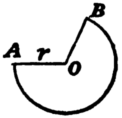
5. Find the volume of a cone in which $r=8$ in. and $h=12$ in. Make a drawing of the cone.

6 Now can you find the volume (in cubic inches) of one of the ice-cream cones referred to at the beginning of this section? How many cubic inches are there in its volume? How many cones could be filled with a gallon of ice-cream?

7. If the ice-cream costs 5*s*. 6*d*. per gallon, and the cones cost $\frac{1}{2}$*d*. each, what is the cost of each filled cone?

8. The diameter of the base of a cone is 14 in.; its altitude is $8\frac{1}{2}$ in. Find its volume.

9. Construct a cone by cutting out a sector (part) of a circle, as in this figure. Place OA along OB.



10. How many loads of sand (27 cu. ft. to the load) are there in a conical-shaped pile of sand, 18 ft. across its base and 8 ft. high?

11· Find the number of bushels of wheat in a conical pile, 12 ft. across the base and $5\frac{1}{4}$ ft. high. (Use 1·24 cu. ft. for 1 bu.)

12. John bought lemonade at 1*s*. 6*d*. a gallon, and sold it in paper conical cups at 3*d*. a cup. How much did he make on a gallon if the diameter of each cup was 4 in. and the depth was 4 in. The cups cost $\frac{1}{2}$*d*. each. (Take 1 gal. $=277$ cu. in.)

13. Find the volume of a cone in which r is $\frac{1}{2}$ ft. and h is 8 in

14. Find the volumes of cones whose dimensions are:

(*a*) $r=10$ in., $h=10$ in.
(*b*) $r=5$ in., $h=10$ in.
(*c*) $r=5$ in., $h=5$ in.
(*d*) $r=1$ cm., $h=3$ cm.
(*e*) $r=1$ cm., $h=6$ cm.
(*f*) $r=2$ cm., $h=6$ cm.

77. Finding the Surface of a Sphere.—The boys in a gymnasium class decided to try to put a new leather cover on their punch-ball. They knew that it was a sphere 30 in. in diameter. Their instructor said: "Find the *surface* of the ball, and then add 10 per cent. for waste in cutting and sewing." Can you calculate the amount of leather the boys had to buy?

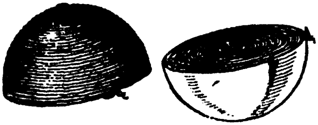
EXERCISES VIII

1. An experiment to find how to calculate the surface of a sphere:

(*a*) Cut a spherical object into two equal parts (hemispheres). A round potato or an old croquet ball will serve your purpose.

(*b*) Wrap a string on the flat surface as suggested in the figure.

(*c*) Wrap a string on the curved surface of one of the hemispheres, as shown here. (You need a heavy cord.)



(*d*) Compare the amount of string used in (*b*) and (*c*).

(*e*) Show that the flat surface contains $3\frac{1}{7} r^2$ square units.

(*f*) If your work was accurately done you used twice as much string on the curved surface as on the flat surface. Show that the surface of the hemisphere, therefore, is $2 \times 3\frac{1}{7} \times r^2$.

(*g*) Show that the *total* surface of the sphere is $4 \times 3\frac{1}{7} \times r^2$ square units.

2. Now find the surface of the punch-ball referred to at the beginning of this section.

3. How much leather will be needed to make a ball which is 10 in. in diameter? Allow 10 per cent. for seams.

4. The radius of one sphere is 4 in.; the diameter of another sphere is 4 in. Compare their surfaces.

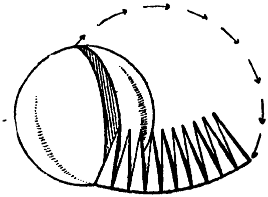
78. To measure the Volume of a Sphere.—Martha, who did the family buying, had a chance to buy oranges, 3 in. in diameter, for 1*s*. 6*d*. a dozen. The grocer urged her to buy oranges 4 in. in diameter at 3*s*. a dozen. She refused to buy the latter. Was Martha a good buyer? Why? (Disregard thickness of the orange skin.)

To be certain about a problem like Martha's you should know how to find the volume of a sphere.

EXERCISES VIIK

1. An experiment to find how to calculate the volume of a sphere :

(*a*) Cut a spherical object into pyramids as suggested by this figure. (An apple or a potato will do.)



(*b*) What is the height of each pyramid ?

(*c*) How would you find the volume of each pyramid ?

The volume of each pyramid equals its base times ——.

(*d*) What is the sum of the areas of all the bases ? Show that this is 4 times $\frac{22}{7}$ times r^2.

(*e*) Show that the sum of the volumes of all pyramids equals the surface of the bases $(4 \times \frac{22}{7} r^2)$ multiplied by one-third of the height.

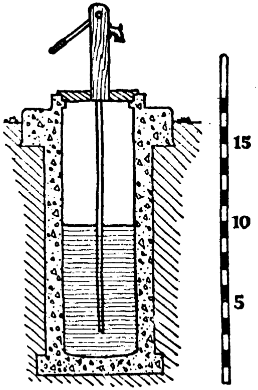
(*f*) Show that the volume of a sphere equals $\frac{1}{3}$ of r times 4 times $\frac{22}{7} r^2$ or that $V = \frac{4}{3} \times \frac{22}{7} r^3$.

2. What is the volume of an orange whose radius is $1\frac{1}{2}$ in. ? One whose radius is 2 in. ?

3. The volume of an orange of 2-in. radius is how many times as large as the volume of an orange of $1\frac{1}{2}$-in. radius ? Was Martha a good buyer ? (See § 78 above.)

4. A glass bowl in the form of a hemisphere 10 in. in diameter will hold —— gallons. (Take 1 gal. = 277 cu. in.)

79. The Well on Mr Goodacre's Farm. — Many farmers get their water from wells. During long, dry summers these wells sometimes run dry. Alfred often saw his father, Mr Goodacre, looking down into the well to see about how deep the water was. He thought of an easy way to tell how much water there was in the well. His idea was to get a long pole, mark foot lengths on it, and then put it down to the bottom of the well. This would show him the *depth* of the water. From this he could easily reckon the number of gallons of water in the well.



Exercises VIII.

1. The diameter of the well was 4 ft. If the water was 1 ft. deep there would be —— cubic feet of water in the well.

2. There are $6\frac{1}{4}$ gal. in a cubic foot of water. For each foot of its depth the water would amount to —— gallons.

3. Make a table showing the quantity of water for each number of feet of depth of the water.

4. How could Alfred have made a gauge that would have shown at a glance the number of gallons of water in the well?

5. Make a sketch of a gauge that will tell the number of gallons of water in a well that is 8 ft. in diameter.

80. How long will the Coal last?—Mrs Avery liked a bright fire on a winter's evening. One winter, when the weather was unusually severe, she said she wondered if the coal in the cellar would last until spring. Fred saw that he could help his mother by showing her that there was enough coal to last them the three remaining cold months. He couldn't weigh the coal, so he had to reckon the number of tons. Fred asked his general science teacher for a table showing the relation between the weights of different substances and that of water. He learned from this table that a cubic foot of soft coal is 1·28 times as heavy as a cubic foot of water.

EXERCISES VIIM

1. If a cubic foot of water weighs 62·5 pounds, what is the weight of a cubic foot of soft coal ?

2. The floor of the Avery coal-cellar was 8 ft. long and 6 ft. wide. Fred levelled down the coal and estimated the average depth of the coal to be 4 ft. How many cubic feet of coal were there in the cellar ?

3. How many pounds of coal were there in the cellar ? how many tons ? (2240 lb. = 1 ton.)

4. If the Avery family use 1 ton per month, will the coal last six months ?

5. The coal-cellar of the Nelson School is 12′ by 10′ by 8′, and is three-fourths full. How many tons of coal has the school ?

6. Mrs Avery paid 42*s*. per ton for coal. She bought three wagonloads weighing 2750 lb., 2600 lb., and 2150 lb. respectively. How much was the coal bill ?

7. Find the cost of 3760 lb. of coal at 49*s*. per ton.

EXERCISES VIIN

Various Problems

1. How many tons of loose hay can be put in a barn 32 ft. long, 14 ft. wide, and 11 ft. high, allowing 512 cu. ft. to the ton ?

2. Mr Miller excavated ground for a cellar. How many loads of earth (1 cu. yd. = 1 load) did he remove if the cellar was 16 ft. long, 12 ft. 5 in. wide, and 6 ft. 3 in. deep ?

3. Could a 14-year-old boy or girl lift a block of ice 30 in. long, 18 in. wide, and 10 in. thick if 1 cu. ft. of ice weighs 57·5 pounds ?

4. John Ford wanted to know the average number of miles he was getting out of a gallon of petrol. He kept this record :

Date	Speedometer Reading	No. Gallons bought
July 1	468	4
5	540	5
8	581	3
10	645	5
14	741	6
15	823	

He estimated that there was as much petrol in the tank when he concluded the record as when he began it. He reckoned that he was getting an average of —— miles per gallon. Do you need to use all the figures in the middle column ?

5. There has been some difference of opinion among scientists concerning the number of inches in a metre.

Here are the opinions of five experts :

In 1817 Hossler said 1 metre = 39·36994 in.
In 1818 Kater said 1 metre = 39·36990 in.
In 1835 Bailey said 1 metre = 39·36973 in.
In 1866 Clarke said 1 metre = 39·36970 in.
In 1885 Comstock said 1 metre = 39·36984 in.

We take the average of these results, or —— inches, as the length of one metre. (This is the length of the standard metre mentioned in Chapter I.)

6. Jane took care of her neighbour's baby one afternoon from half-past one until a quarter to five. At the rate of 8*d*. an hour, how much money did she earn?

7. If apples are selling at the rate of 3 for 5*d*., what would 2¼ dozen cost ?

8. A football team played 28 games during one season and won 18 of them. What per cent. of the games played did the team lose ?

9. Mr Jones earns £1400 a year. His rent is £10 a month, and all of his other expenses amount to £800 a year. He saves the remainder of his earnings. What per cent. of his earnings does he save ?

10. How much will it cost to cover a rectangular kitchen floor 9 ft. by 15 ft. with linoleum at 8*s*. 6*d*. a square yard?

11. Mrs Everyman purchased 4½ yd. of lace at 2*s*. 4*d*. a yard, and ¾ yd. of material at 8*s*. 6*d*. a yard. How much change should she receive out of a £1 note ?

12. Mr Clothier paid £3 15*s*. for a suit that he marked to sell at a profit of 20 per cent. on the cost. Later he sold it at a discount of 20 per cent. of the marked price. Did he gain or lose ? How much ?

13. How many gallons of water could you place in a rectangular tank whose inside dimensions are 3 ft., 2 ft., and 6 in. ? (Take 1 cu. ft. = 6¼ gal.)

14. A motorist left a certain town at 9.45 A.M., driving due west at an average rate of 20 miles an hour. At what

time of day did he arrive at a place 130 miles due west, if he took 30 minutes for lunch ?

15. Mary's watch loses 1 minute and 30 seconds every 24 hours. She set it right on Saturday at noon. What did her watch show at noon on the following Tuesday ?

16. The boys in Sir William Thomson's school had a competition to reduce lateness. Here is the record :

Form	No. of Boys in the Form	No. of 'Lates' in the Month
IB	18	2
IA	19	3
IIB	23	3
IIA	21	2
IIIB	24	4
IIIA	22	2

Which form did best in the competition ? second best ? worst ?

FAMILY BUDGETS

81. Here is a proposed budget for family expenditures :

Annual Income	Per Cent. of Income				
	Food	Rent and Rates	Clothing	Working Expenses	Amusements, etc., and Savings
£500 to £1200 . .	25	20	20	15	20
£300 to £500 . .	30	20	20	15	15
£240 to £300 . .	35	20	15	15	15
£150 to £240 . .	45	15	15	15	10
Under £150 . .	60	10	10	10	10

And here are some questions bearing upon the above budget:

EXERCISES VIIo

1. Can a householder whose income is £400 a year afford to live in a house rented at £64 a year if the rates are 10*s*. in the £ ?

2. Give the exact amounts which should be spent under each of the headings in the above table if the annual income is £750.

3. In a certain family where the annual income is £280 a year, £105 a year is spent on food, £48 on rent and rates, £50 on clothing, £40 on working expenses, and £37 on amusements, etc., and savings How much are these amounts above or below the amounts which the above budget would allow ?

4. Mr Brown draws a salary of £30 a month. He can afford to pay £—— for rent and rates per month, and to spend £—— on clothing for the family each year.

5. It will be interesting to have pupils bring in proposed budgets, and to compare the per cents. which are set aside for the various items of expense.

6. Why should a family which has a small income spend such a large per cent. of it on food ?

7. Reckon the number of pounds each of the following families should spend each month for the items given in the budget. Enter your results in a table like this :

Family	Annual Income	Amount of Money to be spent each Month				
		Food	Rent and Rates	Clothing	Working Expenses	Amusements, etc., and Savings
Hancock .	£700					
Freebody .	£1000					
Lewis . .	£255					
Marshall .	£475					
Pearson .	£130					

8. The Tomkins family had a monthly income of £40. One month their expenses were food, £16; rent and rates, £8; clothing, £6; working expenses, £5; savings, the remainder of the monthly income. How much more (or less) did they spend for food than the budget (p. 193) allowed? How much more (or less) for rent and rates? for clothing? for working expenses? How much was saved, and by how much was this short of the proper amount?

9. A famous railway financier once said that anyone who would save regularly 10 per cent. of his income would eventually be in comfortable circumstances financially. Give some reasons why his statement was, or was not, correct.

Running the Division 'Hurdle' Test

You should practise this test until you can 'take' these hurdles in 15 minutes.

Hurdle 1

(*a*)	(*b*)	(*c*)	(*d*)
$25\overline{)6725}$	$68\overline{)43,452}$	$94\overline{)67,680}$	$89\overline{)32,721}$

Hurdle 2

(*a*)	(*b*)	(*c*)	(*d*)
$14\overline{)56,056}$	468 ÷ 7 =	$6{\cdot}2\overline{)18{\cdot}6}$	$25\overline{)9725}$

Hurdle 3

(*a*)	(*b*)	(*c*)	(*d*)
$0{\cdot}06\overline{)24}$	$0{\cdot}07\overline{)161}$	$0{\cdot}4\overline{)260}$	$0{\cdot}02\overline{)0{\cdot}004}$

Hurdle 4

(*a*)	(*b*)	(*c*)
$\frac{2}{3}$ yd. ÷ 6 = —— in.	$8 \div 12\frac{1}{2} =$	$3\frac{1}{2} \div 9 =$

Hurdle 5

(a)	(b)	(c)	(d)
$\frac{3}{4} \div \frac{1}{2} =$	$1\frac{1}{2} \div \frac{3}{4} =$	$\frac{3}{5} \div 4 =$	1600)40000

Hurdle 6

(a)	(b)	(c)	(d)
3·1)10	0·03)45	0·42)16·8	$4{\cdot}5 \div \frac{3}{4} =$

REVISION EXERCISES VII

1. In the left-hand column below is a list of formulæ. In the right-hand column is a list of subjects which suggest what the formulæ in the first column mean. Match these two columns. You will not need to mark your book if you lay the edge of a sheet of paper along the left-hand column.

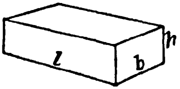
	$V=S^3$	1	Surface of a sphere
	$V=lbh$	2	Volume of a cone
	$V=\frac{4}{3}\pi r^3$	3	Volume of a cube
	$V=\frac{\pi r^2 h}{3}$	4	Volume of a sphere
	$V=\pi r^2 h$	5	Volume of a cylinder
	$S=4\pi r^2$	6	Volume of a pyramid
	$V=\frac{Bh}{3}$	7	Volume of a prism

2. In what way are inches, centimetres, and miles alike ?

3. In what way are square inches, square yards, square metres, and square centimetres alike ?

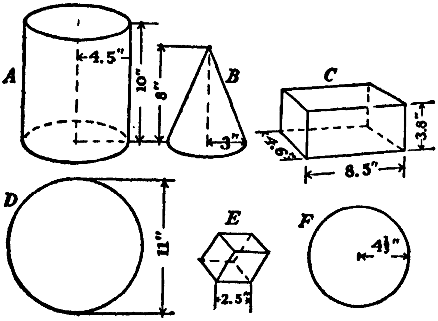
4. In what way are centimetres, square inches, and cubic feet alike ?

5. Write the formula for the volume of the rectangular solid shown here. What is its entire surface if l is 12, b is $8\frac{1}{2}$, and $h = 4{\cdot}6$?



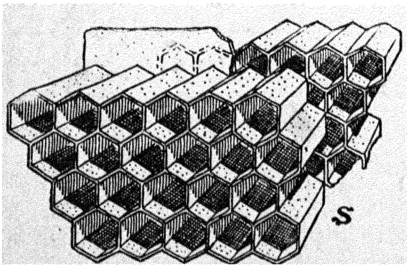
6. Find the surface and the volume of these figures :

Enter your results in a table like the one shown here.



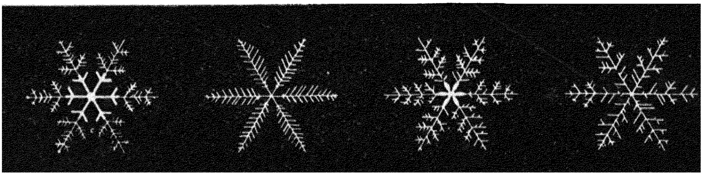
Leave out any part which has not yet been taught. (*D* is a sphere. *F* is a circle.)

Figure	Surface	Volume	Figure	Surface	Volume
A			*D*		[*Omit*]
B	[*Omit*]		*E*		
C			*F*		[*Omit*]



7. This figure was sketched from a piece of honeycomb. What geometrical figure does the honey bee use ? Do you think that square cells would do as well ? In advanced mathematics we learn that cells of no other shape would be as economical.

8. Many interesting geometric forms are found in nature. The figures shown here were drawn from actual snow crystals. Upon what geometric figure are they based? How are they drawn?



PRACTICE FOR ACCURACY AND SPEED

Test 18. Addition of Fractions

Reduce your results to lowest terms.

(1) $\frac{1}{2}+\frac{2}{3}+\frac{1}{4}=$ (5) $\frac{1}{2}+\frac{3}{8}+\frac{1}{4}=$ (9) $\frac{1}{3}+\frac{3}{4}+\frac{5}{12}=$

(2) $\frac{1}{3}+\frac{1}{4}+\frac{5}{6}=$ (6) $\frac{1}{2}+\frac{3}{10}+\frac{1}{5}=$ (10) $\frac{2}{3}+\frac{1}{2}+\frac{5}{6}=$

(3) $\frac{1}{6}+\frac{3}{8}+\frac{1}{12}=$ (7) $\frac{1}{2}+\frac{1}{8}+\frac{5}{12}=$ (11) $\frac{2}{3}+\frac{1}{4}+\frac{3}{8}=$

(4) $\frac{1}{2}+\frac{3}{4}+\frac{1}{6}=$ (8) $\frac{1}{2}+\frac{1}{16}+\frac{3}{8}=$ (12) $\frac{3}{4}+\frac{1}{5}+\frac{3}{10}=$

Test 19. Multiplication of Fractions

(1) $\frac{3}{4}\times 60=$ (3) $\frac{3}{4}\times\frac{1}{2}=$ (5) $56\times\frac{4}{7}=$

(2) $\frac{2}{3}\times 20=$ (4) $2\frac{1}{2}\times 3\frac{1}{2}=$ (6) $3\frac{1}{4}\times 2\frac{1}{2}=$

(7) $\frac{1}{3} \times 9\frac{1}{2} =$

(8) $\frac{2}{3} \times 25 =$

(9) $\frac{3}{8} \times 50 =$

(10) $\frac{4}{5} \times 12 =$

(11) $\frac{3}{8} \times \frac{4}{9} =$

(12) $1\frac{1}{2} \times 6 =$

(13) $2\frac{1}{2} \times 4\frac{1}{2} =$

(14) $2\frac{2}{3} \times 9 =$

(15) $6 \times 3\frac{3}{4} =$

Test 20. Division of Fractions

(1) $\frac{3}{4} \div \frac{1}{4} =$

(2) $\frac{3}{4} \div \frac{1}{8} =$

(3) $\frac{3}{2} \div \frac{1}{4} =$

(4) $\frac{1}{2} \div \frac{1}{3} =$

(5) $\frac{1}{2} \div 2 =$

(6) $\frac{5}{8} \div \frac{1}{2} =$

(7) $\frac{5}{12} \div \frac{2}{3} =$

(8) $6 \div \frac{2}{3} =$

(9) $\frac{1}{8} \div \frac{1}{8} =$

(10) $\frac{1}{4} \div \frac{1}{8} =$

(11) $\frac{1}{3} \div \frac{1}{2} =$

(12) $\frac{3}{4} \div \frac{1}{5} =$

(13) $\frac{3}{5} \div \frac{3}{4} =$

(14) $\frac{7}{2} \div \frac{1}{2} =$

(15) $4 \div \frac{4}{3} =$

When you pass Test 20 turn to Test 21, p. 207.

CHAPTER VIII

"TAKE CARE OF THE PENCE"
PERSONAL ACCOUNTS, BILLS, ETC.

82. "Take Care of the Pence."—Every one should learn early in life to put by a little money each week. An ordinary money-box may be used, or better still a start should be made in the Post Office Savings Bank, where in time interest will be allowed and the money will begin to grow! You will be surprised to learn later how money does grow.

Here is a striking example of the growth of money. Note it carefully, and perhaps it will make you determined to save a little each week, and to bank what you save.

Suppose that James I when he became King of England in 1603 had banked £1 in a bank which from his day to ours paid 2½ per cent. interest per annum. Further, suppose he had said: "The money is not to be withdrawn from the bank till 1903, when whoever is King of England in that year may withdraw it and have it for his own." How much do you think Edward VII in 1903 might have withdrawn from the bank if the above had been true? *A little more than* £1620!

In working out some of the following exercises you will also learn how money grows and how wise it is to save all you can.

EXERCISES VIIIA

1 Mr and Mrs Hartley decided at the time of their daughter Mary's birth to put by 2*s*. 6*d*. a week, and to give their daughter on her twenty-first birthday the total amount then saved. How much did Mary get? (Do not reckon interest.)

2. A father said to his son on the latter's tenth birthday: "To every penny you save out of your pocket-money I will add twopence from now until your eleventh birthday." These are the amounts the boy saved during each of the next twelve months: 1*s*. 4*d*., 2*s*. 1*d*., 3*s*. 2*d*., 7*s*. 1*d*., 1*s*. 4½*d*., 1*s*., 5*s*. 1½*d*., 4*s*. 3*d*., 2*s*. 7*d*., 6*s*. 2*d*., 1*s*. 6*d*., 2*s*. How much had the boy altogether when his father had fulfilled his promise?

3. A father said to his son: "Every month you save more than 5*s*. I will add half the amount you save; every month you save less than 5*s*. I will add one-quarter of the amount you save." These are the amounts the boy saved per month for a year: 5*s*. 10*d*., 3*s*. 8*d*., 2*s*. 4*d*., 7*s*. 6*d*., 6*s*. 0*d*., 1*s*. 10*d*., 4*s*. 8*d*., 3*s*. 2*d*., 5*s*. 6*d*., 5*s*.10*d*., 2*s*. 6*d*., 5*s*. 1*d*. How much had the boy altogether when his father had fulfilled his promise?

4. A boy wished to buy a camera the price of which was £3 10*s*. He saved 6*d*. a day (including Sundays). How long did it take him to save enough to get the camera?

5. Mr Palmer smoked 4 cigars a day. He bought the cigars at 10*s*. 6*d*. a dozen. He decided to give up smoking and save the money. How much would he save in 10 years? (Do not count interest.)

6. If Kathleen Tanner begins at the age of 14 years to

save 2*d*. a day for 320 days a year, and deposits the money in a bank, how much will she have when she is twenty years old, not counting interest?

7. Mr and Mrs Kerr decided at the time of their son Sidney's birth that in order to give him a start in life at 18 years of age they would bank for him on each of his birthdays twice as many pounds as the number of years he had then lived. How much had he to start with in life?

8. The following table gives the amount of pocket-money that certain boys had each week and the amount each saved. Which boy was the best saver? second best? worst? Fill in the per cent. column.

Boy	Amount of Pocket-money a Week	Amount Saved	Per Cent. Saved
Peter	1*s*. 6*d*.	9*d*.	
Frank	2*s*. 0*d*.	8*d*.	
Albert	3*s*. 0*d*.	1*s*. 3*d*.	
Fred	1*s*. 0*d*.	8*d*.	
Jack	2*s*. 6*d*.	1*s*. 0*d*.	
Edward	3*s*. 6*d*.	2*s*. 6*d*.	

83. On keeping Accounts.—All boys and girls should learn early in life to keep accounts—*i.e.*, they should keep a record of what money they receive and how much they spend.

EXERCISES VIIIB

1. Ernest Potter made in his diary the following entries during the month of October: "Oct. 1, from Dad, 2*s*. 6*d*.," "Oct. 3, pencil, 4*d*.," "Oct. 7, sweets, 6*d*.," "Oct. 10, from Uncle, 1*s*.," "Oct. 15, story-book, 2*s*.," "Oct. 17, sweets, 6*d*.," "Oct. 18, sold my knife, 1*s*.," "Oct. 25, fireworks, 9*d*.," "Oct. 29, from Dad, 2*s*. 6*d*.," "Oct. 31, foreign

stamps, 2*s*. 1*d*." Make out for him his accounts for the month of October.

2. Make out the accounts that a boy of thirteen would be likely to have for the month of August. Suppose he spends the month at the seaside and has 2*s*. a week pocket-money.

3. Here are the income items and expenditure items of a school football club. Draw up a balance-sheet for them. Income: 200 2*s*. 6*d*. subscriptions; grant from school funds, £15. Expenses: travelling, £17 14*s*. 6*d*.; fixture cards, £2; footballs and goal-nets, £8 17*s*. 6*d*.; teas for visiting teams, £5 11*s*. 3*d*.; preparation and up-keep of ground, £3 4*s*.

4. A lady took £2 to market, and spent the following sums in cash purchases: meat, 10*s*. 11*d*.; butter, 5*s*. 6*d*.; eggs, 2*s*. 3*d*.; bacon, 3*s*. 5*d*.; sundry vegetables and fruit, 7*s*. 1*d*. Make out her accounts for her.

5. A gentleman goes shopping with £3 in his pocket. He spends the following sums in cash purchases: collars, 6*s*.; ties, 9*s*.; books, 14*s*. 3*d*.; tobacco and cigarettes, 14*s*. 7*d*.; a present for his wife, 12*s*. 6*d*. Make out his accounts for him.

6. John Smith, a salesman, received £4 10*s*. a week fixed salary and a 10 per cent. commission on sales. During a certain month the value of the goods he sold each week was: 1st week, £53; 2nd week, £34; 3rd week, £29; 4th week, £46. During the same four weeks he spent: 1st week, £7 14*s*. 3*d*.; 2nd week, £6 15*s*. 6*d*.; 3rd week, £6 18*s*. 4*d*.; 4th week, £7 1*s*. 5*d*. Make out his accounts for the month.

84. Bills.—When your mother goes shopping and makes many purchases at one shop, even if she pays cash, she is careful always to have a receipted bill. This is a help to her in doing her accounts. When a person does not pay at the time of purchase for goods bought, the shop or firm sends him a bill at the end of the week or the month or the quarter.

Some shops give discount for cash payments—*i.e.*, 5 per cent. or 10 per cent. of the total of the bill is taken off. And conversely some shops charge interest on overdue accounts—*i.e.*, 2½ per cent. or 5 per cent. is added on to the total of the bill.

EXERCISES VIIIc

1. Make out a bill for the following :

   4 note-books @ 3*d.* each ; 3 penholders @ 1*d.* each; 2 bottles of ink @ 2*d.* each ; 4 rulers @ 2½*d.* each.

2. Make out a bill for the following :

   1 doz. English text-books @ 2*s.* 6*d.* each ; 1½ doz. history text-books @ 3*s.* each ; 1¼ doz. geography text-books @ 2*s.* 3*d.* each.

3. Make out a bill for the following :

   6 lb. of sugar @ 4*d.* a lb. ; 3 lb. of coffee @ 2*s.* 6*d.* a lb. ; 2 lb. of butter @ 2*s.* 4*d.* a lb. ; 3½ lb. of bacon @ 1*s.* 8*d.* a lb.

4. Make out a bill for the following :

   1½ doz. collars @ 10*d.* each ; 4 ties @ 4*s.* 6*d.* each ; 1 doz. handkerchiefs @ 1*s.* 1*d.* each ; 2 flannel shirts @ 15*s.* 6*d.* each.

5. Make out a bill for the following :

   25 cigars @ £4 17*s.* 8*d.* a 100 ; 250 cigarettes @ 6*s.* 11*d.* a 100 ; 3½ lb. of tobacco @ 17*s.* 6*d.* a lb.

6. Make out a bill for the following groceries bought for a large hotel (10 per cent. discount for cash paid at time of purchase).

   5 doz. lb. of sugar @ 4*d.* a lb. ; 56 lb. of butter @ 2*s.* 4*d.* a lb. ; 56 lb. of bacon @ 1*s.* 8*d.* a lb. ; 4 doz. lb. of coffee @ 2*s.* 6*d.* a lb. ; 6 doz. lb. of tea @ 2*s.* 8*d.* a lb.

7. Make out a bill for the following :

28 lb. of apples @ 9*d.* a lb. ; 56 lb. of apples @ 5*d.* a lb. ; 4½ doz. bananas @ 1*s.* 6*d.* a dozen ; 1½ doz. oranges @ 1½*d.* each ; 1 doz. lemons @ 2½*d.* each.

(5 per cent. discount for cash.)

8. Make out a bill for the following :

½ doz. cricket balls @ 11*s.* 6*d.* each ; ½ doz. cricket balls @ 8*s.* 6*d.* each ; 3 cricket bats @ 37*s.* 6*d.* each ; ½ doz. cricket bats @ 19*s.* 6*d.* each.

(5 per cent. discount for cash.)

9. Make out a bill for the following :

6 chairs @ 25*s.* 9*d.* each ; 2 chairs @ 37*s.* 6*d.* each ; 1 table, £4 4*s.* ; 1 sideboard, £8 19*s.* 6*d.* ; 2 small tables @ 24*s.* each.

(5 per cent. discount for cash.)

10. Make out a bill for the following :

6 doz. rulers @ 2*d.* each ; 5 doz. sets of mathematical instruments @ 2*s.* 6*d.* each ; 1 gross of pencils at 10*d.* a dozen ; 1 gross of penholders @ 9*d.* a dozen ; 4 doz. indiarubbers @ 1*s.* 9*d.* a dozen.

(5 per cent. discount for cash.)

85. Discount.—We saw in the paragraph above that the amount taken off a bill for cash payment is called discount. Discount is allowed for other reasons as well—*e.g.*, sometimes employees in a shop are allowed to purchase goods at that shop for less than the marked price. In this and in other ways the price of a certain article may be subjected to two discounts. The second discount is always reckoned on what is left after the first discount has been deducted. The net price of an article is what is actually paid for it after all discounts have been deducted.

Exercises VIIId

1. From £100 take 10 per cent., and then 5 per cent. from the remainder.

2. From £1000 take 20 per cent., and then 1 per cent. from the remainder.

3. From £750 take $33\frac{1}{3}$ per cent., and then 10 per cent. from the remainder.

4. Find the net price of a motor-car marked £850, with discounts of 10 per cent. and 10 per cent.

5. A piano is priced at £90. What is its net price with discounts of 15 per cent. and 10 per cent.?

6. Find the net price of goods marked and discounted as follows:

(*a*) £30, 30 per cent., 20 per cent.
(*b*) £20, 35 per cent., 5 per cent.
(*c*) £42, 25 per cent., $12\frac{1}{2}$ per cent.
(*d*) £7 10*s*., 15 per cent., 10 per cent.
(*e*) £7 10*s*., 10 per cent., 15 per cent.
(*f*) £90, 50 per cent., 50 per cent.

7. Which would you prefer if you were buying a gramophone marked £25: discounts of 10 per cent. and 10 per cent. or a single discount of 20 per cent.?

8. Which is better for the buyer, a single discount of 25 per cent. or discounts of 15 per cent. and 10 per cent.?

9. As a buyer, would you prefer a 10 per cent., 20 per cent. discount or a 20 per cent., 10 per cent. discount.

10. Which is better for the buyer, a single discount of 35 per cent. or discounts of 30 per cent. and 5 per cent.?

Revision Exercises VIII

1. Select the correct parts in each of the following:

(*a*) $\frac{2}{3}=\frac{20}{30}=66\frac{2}{3}$ per cent. $=\frac{66\frac{2}{3}}{100}=0\cdot66=2$ out of 3.

(*b*) 10 per cent. $=0\cdot1=\frac{10}{100}=\frac{100}{1000}=\frac{1}{10}=\frac{0\cdot10}{100}=$ ratio of 1 to 10.

(*c*) $\frac{3}{2}=$ ratio of 3 to $2=\frac{150}{100}=150$ per cent. $=1\cdot5=\frac{15}{100}$.

2. Complete each of these statements :

(*a*) An inch is —— per cent. of a foot.
(*b*) A foot is —— per cent. of a yard.
(*c*) An ounce is —— per cent. of a pound.
(*d*) A pint is —— per cent. of a quart.
(*e*) A quart is —— per cent. of a gallon.
(*f*) A peck is —— per cent. of a bushel.
(*g*) A centimetre is —— per cent. of a metre.
(*h*) A pound is —— per cent. of a ton.
(*i*) A florin is —— per cent. of a pound sterling.
(*j*) A sixpence is —— per cent. of five shillings.
(*k*) Six shillings and eightpence is —— per cent. of a pound sterling.
(*l*) An hour is —— per cent. of a day.

3. Complete each of the following :

(*a*) 60 is 50 per cent. of ——.
(*b*) 12½ is 25 per cent. of ——.
(*c*) 40 is 200 per cent. of ——.
(*d*) 5 is ½ per cent. of ——.
(*e*) 73 days is 20 per cent. of ——.
(*f*) 1¼ yd. is 25 per cent. of ——.

4. Find the rate of discount on the following :

(*a*) A set of dishes marked at 40*s*. and bought at 30*s*.
(*b*) A pair of shoes marked down from 40*s*. to 25*s*.
(*c*) Handkerchiefs marked down from 8*s*. a dozen to 6*s*. a dozen.
(*d*) A coal-scuttle marked 6*s*. 8*d*. and sold for 5*s*.

PRACTICE FOR ACCURACY AND SPEED

Test 21. Operations with Fractions

(1) $\frac{1}{2}+\frac{1}{3}=$
(2) $\frac{1}{2}-\frac{1}{3}=$
(3) $\frac{1}{2}\div\frac{1}{3}=$
(4) $\frac{1}{2}\times\frac{1}{3}=$
(5) $\frac{5}{8}+\frac{1}{4}=$
(6) $\frac{5}{8}-\frac{1}{4}=$

(7) $\frac{5}{8} \div \frac{1}{4} =$

(8) $\frac{5}{8} \times \frac{1}{4} =$

(9) $\frac{2}{3} \times \frac{3}{4} =$

(10) $\frac{4}{5} - \frac{2}{3} =$

(11) $\frac{3}{8} \times \frac{4}{5} =$

(12) $6 \div \frac{3}{4} =$

(13) $\frac{1}{6} + \frac{3}{8} =$

(14) $6 \div \frac{1}{6} =$

(15) $\frac{2}{3} \times \frac{3}{2} =$

Test 22. Mixed Exercises on the Fundamental Rules

(1) Add : $\begin{array}{r} 7 \\ 9 \\ \hline \end{array}$

(2) $7\overline{)60}$

(3) $\begin{array}{r} 0{\cdot}27 \\ \times 3 \\ \hline \end{array}$

(4) Subtract : $\begin{array}{r} 41 \\ 17 \\ \hline \end{array}$

(5) $7 + 19$

(6) $\begin{array}{r} 276 \\ \times 8 \\ \hline \end{array}$

(7) $19 \div 9$

(8) $17 + 9 + 34 + 7$

(9) $4 - 2\frac{1}{4}$

(10) Add : $\begin{array}{r} 7 \\ 16 \\ 4 \\ 18 \\ \hline \end{array}$

(11) $7\overline{)1393}$

(12) Subtract : $\begin{array}{r} 63 \\ 9\frac{1}{4} \\ \hline \end{array}$

(13) $\frac{2}{3} \div 4$

(14) $\frac{3}{4}$ of 19

(15) $8{\cdot}2 - 2{\cdot}002$

(16) $4\overline{)9 \text{ lb. } 4 \text{ oz.}}$

MISCELLANEOUS PROBLEMS

1. The children of a certain school were raising money to buy equipment for their gymnasium. Mr Archer, a friend of the school, said : " I will give in the ratio of 5 to 2 that you give." The children understood that he would give 2*s*. 6*d*. for every shilling they raised. Were they right ? If the children raised £8 then Mr Archer would give £——.

2. In making a gargle Elizabeth mixes peroxide with hot water in the ratio of 1 to 3·5. How many tablespoonfuls of water does she put with 3 tablespoonfuls of peroxide ?

3. How many times as much will it cost :

(*a*) For 16 gallons of petrol as for 12 gallons of petrol ?

(*b*) For a carpenter for 36 days as for a carpenter for 30 days ?

(*c*) For bus fares for 30 people (all adults) as for bus fares for 12 people ?

4. Mr Saunders, a wholesale fruiterer, examined 18 baskets of strawberries out of a lot of 260 baskets. 6 of the 18 were not fit to sell. How many of the 260 boxes were probably not fit to sell ?

5. The Randwick Rovers played 29 matches during the season, and won 17 of them. What part of the whole number of games did the team win ? What per cent. ?

6. Mr Nugent receives a salary of £1400 a year. His rent costs him one-tenth of this and his other expenses are £900. He saves the rest. What per cent. of his salary does he save ?

7. A tank held $40\frac{3}{16}$ gallons of oil. One day $16\frac{3}{4}$ gallons

were drawn out, and the next day $8\frac{1}{8}$ gallons. How many gallons were left in the tank ?

8. Mrs A., Mrs B., and Mrs C. bought a box of dried peaches. Mrs A. paid 16*s*. 8*d*., Mrs B. paid 13*s*. 4*d*., and Mrs C. paid 10*s*. The peaches weighed 48 lb. How many pounds should each of the ladies have ?

9. On the average one ragweed plant has 2300 seeds. Supposing that every seed grew to be a plant, how many square feet of land would the great-grandchildren of the ragweed plant cover, counting 8 plants to a square foot ? How many acres would they cover ?

10. Compare the good and the harm done to a community by a hawk, supposing that the hawk kills 5 chickens worth 1*s*. 6*d*. each and 812 mice that would each do $\frac{1}{4}$*d*. worth of harm.

11. An ordinary pint cup is 3 in. high ; its circumference is 7 in. How many square inches of tin are there in the bottom of the cup ? How many square inches are there in the curved surface of the cup ?

Fred said that it takes twice as much tin to make a quart cup with the same circumference as a pint cup. Was he correct ?

12. Receipts from the church bazaar were £160. They were divided among the building fund, the music fund, and the missionary fund in these proportions : 5, 3, and 2. How much money did each fund receive ?

13. How much vinegar would Mary use with 8 tablespoonfuls of oil in making salad dressing, when the recipe tells her to "Use 2 parts of oil and 1 part of vinegar" ?

14. Alice's dress requires 2 yd. of velveteen 40 in. wide. About how much will be required if a cloth 48 in. wide is used, if this cloth cuts to fit the pattern as well as the 40-in. width ?

15. Mrs Norton used 6 qt. of blackberry juice and 4 qt. of sugar syrup to make blackberry jelly. She filled 30

½-pt. jars with jelly. How much of her material boiled away or stuck to the preserving-pan ?

16. Assuming that chickens, as purchased in the market, contain 36 per cent. waste, how much edible meat do you obtain from a 9-lb. chicken ? What does the edible meat cost per lb. when the market price of the chicken is 1*s.* 1*d.* per lb. ?

17. A pile of wood 8 ft. long and 4 ft. high and 4 ft. wide is one cord. How many cords are there in a pile 60 ft. long, 4 ft. wide, and 6 ft. high ?

18. At 34*s.* per cord, what is the total value of three piles of 4-ft. wood, each 4 ft. high, if one pile is 8 ft. long, another 30 ft. long, and the third 42 ft. long ?

19. Last year Jane raised chickens. She lost 20 per cent. of all that hatched. How many of the 95 that were hatched did she raise ?

20. £20,000 was required to float a company. £32,640 was invested in shares of varying amounts. This was what per cent. of the sum required ? What per cent. above the sum required ?

21. Milk is about 4 per cent. fat. How many pounds of milk will it take to give 10 lb. of fat ? 18 lb. of fat ? 30 lb. of fat ?

22. How much milk which contains 3·6 per cent. butter fat will be needed for 40 lb. of butter fat ? 60 lb. of butter fat ?

23. It is estimated that 7½ per cent. of the money spent on food is wasted. At this rate how much do the Browns waste per year if their monthly expenditure for food is £12 ? the Smiths, who spend £15 per month ?

24. A book on home architecture states that in a pleasing living-room the width should be 75 per cent. of the length. Write this as a formula using w and l. What is l if w is 18 ?

25. In a well-lighted room the window space should be at least 18 per cent. of the floor space. How large should the window space be in a room 12 ft. by 16 ft. ? 9 ft. by

12 ft. ? 18 ft. by 24 ft. ? Calculate the ratio of the window space of your schoolroom to the floor space. Express this ratio as a percentage.

26. If for doing odd jobs you were offered your choice of taking 16 eggs out of every 50 or 14 out of every 40, which would you choose ?

27. Mr Stephens owned $\frac{5}{8}$ of a factory, and sold $\frac{2}{5}$ of his share. What part of the total value of the factory did he sell ? What part of the factory did he still own ?

28. Make out a bill for Mrs Holmes. She bought :

> 2 lb. of lard @ 1*s*. a lb. ; $1\frac{1}{2}$ lb. of cheese @ 1*s*. 4*d*. a lb. ; $2\frac{1}{2}$ lb. of butter @ 2*s*. 1*d*. a lb. ; 3 lb. of margarine @ $10\frac{1}{2}$*d*. a lb.

29. Mrs Harper bought the following :

> 3 beetroots @ 2*s*. 6*d*. a dozen ; 2 lb. of tomatoes @ 7*d*. a lb. ; 3 cauliflowers @ $4\frac{1}{2}$*d*. each ; 4 lb. of turnips @ $10\frac{1}{2}$*d*. a stone.

Make out a bill.

30. The headmaster of a school purchased the following:

> $1\frac{1}{2}$ gross of exercise-books @ 2*s*. 6*d*. a dozen ; $2\frac{1}{2}$ gross of exercise-books @ 1*s*. 9*d*. a dozen ; 1 gross of exercise-books @ 3*s*. 4*d*. a dozen ; 2 gross of small note-books @ 1*s*. 2*d*. a dozen.

Make out a bill.

31. The difference between what you receive and what you pay does not tell how well you are doing in business. The time and labour spent on the business must be taken account of also. For example, a man kept a bookshop. He paid £212 a year for rent, heat, light, telephone, etc., and hired two clerks at £10 a month each to help him run the business. During one year he bought goods for £1450 and sold them for £2130. How much better off would he have been if he had worked for somebody for £20 a month ?

32. Arthur bought a dog one day for 3 guineas, and

spent an hour a day for 8 days trying to sell it. Food for the dog cost him 5*d.* a day, and he spent 5 hours caring for the dog. He sold the dog for £3 15*s.* How much did he receive per hour for the time spent ?

33. Frank bought hens for 25*s* and sold them at an advance of 20 per cent., spending 2*s.* 1*d.* in bus fares and 5 hours of time. What was his profit per hour ?

34. Jack saw a dog for sale, bought it for £2 10*s.*, and sold it to Mr Hudson for 50 per cent. more than he paid for it. If it took him 3 hours to find the buyer, how much did he make per hour for the time spent ?

35. Fred Richards bought 5 gallons of ice cream at 7*s.* 6*d.* a gallon, and cones for 5*s.*, to sell ices at the school garden-party. Find what per cent. of the money invested he lost (neglect the time spent) :

(*a*) If he sold only 110 ice-cream cones at 3*d.* each and had to give away or throw away all the rest.

(*b*) If he sold 80 cones at 3*d.* each, 20 at 2*d.* each, and 10 at 1½*d.* each, and had to give away all the rest.

(*c*) If it rained so that he could not sell any, but succeeded in persuading the man, from whom he bought the goods, to take them back at 60 per cent. of their cost.

36. Joyce bought 12 dozen oranges at 1*s.* 3*d.* a dozen, less 10 per cent. for cash. She sold 80 of them at 2*d.* each, and the rest at 4 for 7*d.* How much profit did she make ?

37. Mr Webster could earn £1000 a year as a departmental manager in a big hardware company. He keeps a shop of his own instead. Last year he bought goods for £5260 and sold them for £6990. He paid £825 for the expenses of running the shop, not counting any salary for himself. How much less did he earn than he would have received if he had worked for some one else at a salary of £1000 a year ?

38. Mrs Evans is a rich lady who has poor taste in buying hats, so she hires a professional buyer to buy hats

for her. She pays her 5 per cent. for buying. How much did the professional buyer receive as commission for buying £13 worth of hats?

39. Jane sells two magazines. One costs 12*s*. a year, the other costs 14*s*. a year. She receives 5*s*. in the £ from the first magazine and 6*s*. in the £ from the second magazine. Last summer she received 80 subscriptions to the first magazine and 30 to the second magazine. How much did she make on both? How much did she make per hour if she had spent 240 hours in all?

40. If it cost £70 to fence a field 40 rods long and 30 rods wide how much will it cost to fence a field 90 rods square?

41. A piece of land 2 miles square will make how many 80-acre farms?

42. Thirty-two pupils and their teacher occupy a schoolroom 30 feet in length, 26 feet in width, and 15 feet in height. What is the average number of cubic feet of air for each person? Try to find out what laws, if any, concerning air space were kept in mind in the construction of your school building. What is the average number of cubic feet of air space per pupil in your classroom?

43. How much profit is made on 6 dozen pounds of sweets bought at 1*s*. 8*d*. a lb. and sold at 2*d*. an ounce?

44. A rainfall of half an inch amounts to how many gallons of water to the acre? to how many tons?

45. If sound travels at the rate of 1080 ft. a second, how far distant is a thunderstorm if the thunderclap is heard 6 seconds after the lightning flash is seen? The speed of light is so great that it is negligible here. (It is 186,000 miles per second.)

46. If the fall in the barometer is 0·1 in. for every 100 ft. of elevation what is the fall due to an increase in elevation of 2400 feet? of 6400 feet?

47. If a poorly arranged kitchen forces Mrs Shaw to

take 120 steps, 22 in. to the step, each day in preparing meals more than she would in a well-arranged kitchen, how many miles does she travel unnecessarily in the course of a year ?

48. A farmer uses $10\frac{1}{2}$ bu. of grain to sow 5 acres. At this rate, how many bushels are necessary to sow 24 acres ?

49. Two men buy a boat ; one furnishes $\frac{3}{5}$ of the cost, the other $\frac{2}{5}$ of the cost. By letting it they earn £12 10*s*. How should they divide their income ?

50. A bankrupt grocer settled with his creditors, paying 54 per cent. of what he owed. How much did he pay to Alder, to whom was due £150 ? to Ellis, if he owed him £215 ? How much would Alder and Ellis have received if the grocer had settled for 8*s*. in the £ ?

51. A house 40 ft. high had a garage 12 ft. high built along its side. If the height of the house is represented in a drawing by a line 6 in. long, how long a line will be needed to represent the height of the garage ?

52. A train runs from Kemble to London, 90 miles, in 2 hr. 5 min. What is the rate per hour ?

53. Three men rent a pasture for £12. A. puts in 8 cows, B. puts in 10, and C. puts in 12. How much should each man pay ?

54. How many bushels of wheat will a bin hold that is 18 ft. long, 8 ft. wide, and 5 ft. deep, if we allow $1\frac{1}{4}$ cu. ft. to the bushel ?

55. A table is 2′ 6″ wide and 3′ 8″ long. What is the distance round it ?

56. A gallon of ice cream is divided equally among 30 children. What part of a pint does each receive ?

57. A farmer had a cask of sweet cider containing 40 gal. At various times the following were drawn out : 6 gal. 1 qt. 1 pt. ; 7 gal. 1 qt. ; 8 gal. 1 pt. How many gallons of sweet cider were left ?

58. Dr Foster stated that during the first five years of

his practice he earned £700, £830, £1100, £1200, and £1220 respectively. What were his average annual earnings ?

59. May's home is half-way between the school and the railway-station. The latter is $\frac{5}{8}$ of a mile from the school. How many rods is it from her home to the school ?

60. How large an angle do the hands of the clock make at 8 o'clock ? at 2 o'clock ? at 5 o'clock ?

61. How many pounds and ounces of meat are necessary for 10 men each receiving 5 oz. ?

62. My horse, Black Bess, on an average travels a mile in 10 min. 15 sec. How far does she go in 3 hr. ?

63. Express in Roman numerals the number of this question, the number of this page, the present year.

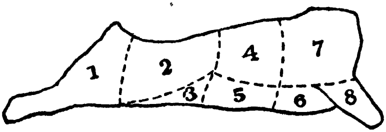
64. Mr Cookson asked Edgar to make him a bookcase for a set of books consisting of 24 volumes. How long must Edgar make each of the two shelves if the books average $2\frac{5}{8}$ in. in thickness ?

65. One month the Wolsey School decreased its number of cases of lateness to 120, from 160 for the previous month. What was the per cent. of decrease ?

66. A joint of meat weighing $5\frac{1}{2}$ lb. cost 8*s*. $8\frac{1}{2}$*d*. How much was the cost per pound ?

67. In the meat industry practically all parts of a beast are used. From the non-edible materials are made such products as leather, glue, soap, and fertilizers.

The carcass of a beast is divided into eight different cuts, as shown here. The approximate per cents. of the dressed weight included in each cut of the beast are given in the table.



1. Round, 24 per cent.
2. Loin, 16 per cent.
3. Flank, 3 per cent.
4. Rib, 10 per cent.
5. Navel, 8 per cent.
6. Brisket, 6 per cent.
7. Chuck, 22 per cent.
8. Shank, 7 per cent.

The dressed carcass of a steer weighed 640 lb. Find the weight included in each of the different cuts for the carcass, using the per cents. given in the table.

68. A hog loses 25 per cent. to 35 per cent. of its weight in being dressed. How much will a 240-lb. hog lose in weight in being dressed if the loss is 32 per cent.? How much will the 240-lb. hog weigh when dressed?

69. The manager of the Haliford Metal Company receives a yearly salary of £1450. He estimates that he works, on an average, 5 hours a day except Sundays. What does he earn per hour?

70. A tradesman buys 40 bicycles at £10 each. He sold three-quarters of them at £12 12*s*. each, 4 of them at £12 each, and the rest, at the end of the season, at £11 11*s*. each. What profit did he make?

71. A railway journey costs 14*s*. 11*d*. How many times must a man make the journey before he has spent £50?

72. A newspaper charges 1*s*. 6*d*. for a single advertisement; for the same advertisement for 6 consecutive days the charge is 5*s*. 6*d*. How much per cent. reduction does the newspaper make?

RECORD OF MY PROGRESS IN MATHEMATICS

Name............................ School...........................

Test	Number of Trials taken to pass	Date when passed	Test	Number of Trials taken to pass	Date when passed
1			12		
2			13		
3			14		
4			15		
5			16		
6			17		
7			18		
8			19		
9			20		
10			21		
11			22		